PRICE ELASTICITIES OF HOUSING SUPPLY

C. PETER RYDELL

R-2846-HUD

SEPTEMBER 1982

HOUSING ASSISTANCE SUPPLY EXPERIMENT

Sponsored by

The Office of Policy Development and Research
U.S. Department of Housing and Urban Development

Rand
SANTA MONICA, CA. 90406

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PREFACE

This report is one of a series analyzing the effect of a shift in the demand for housing services on the price of those services. The other reports focus on price increases caused by specific housing assistance programs. The present report analyzes price increases caused by demand shifts in general.

The other Rand publications in the series are C. Peter Rydell, *Effects of Market Conditions on Prices and Profits of Rental Housing* (P-6008, September 1977); C. Lance Barnett, *Expected and Actual Effects of Housing Allowances on Housing Prices* (P-6184, January 1979); James P. Stucker, *Rent Inflation in Brown County, Wisconsin, 1973-78* (N-1134-HUD, March 1981); C. Lance Barnett and Ira S. Lowry, *How Housing Allowances Affect Housing Prices* (R-2452-HUD, September 1979); D. Scott Lindsay and Ira S. Lowry, *Rent Inflation in St. Joseph County, Indiana, 1974-78* (N-1468-HUD, November 1980); C. Peter Rydell, *Supply Response to the Housing Allowance Program* (N-1338-HUD, October 1980); C. Peter Rydell, John E. Mulford, and Lawrence Helbers, *Price Increases Caused by Housing Assistance Programs* (R-2677-HUD, October 1980); and C. Peter Rydell, Kevin Neels, and C. Lance Barnett, *Price Effects of a Housing Allowance Program* (R-2720-HUD, September 1982).

Charlotte Cox edited the present report. Mayda Redfield typed the draft, and Beverly Westlund produced the final copy.

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SUMMARY

In the long run, a housing market responds to increases in the demand for housing services by using existing residential land more intensely or by increasing the amount of residential land. Both methods of increasing the inventory of housing services raise the average cost of producing those services. The long-run price elasticity of supply is the ratio of relative changes in the inventory of housing services to relative changes in the price of housing services necessary to cover production costs.

The long-run supply of housing services is very elastic, having an estimated long-run price elasticity of 11.5. Most (11.3) of the elasticity is due to inventory changes; a small part (0.2) is due to the upgrading of existing housing by repairs.

In the short run, the housing inventory is fixed (by definition). However, the occupied inventory changes in response to demand shifts. New households occupy dwellings that would otherwise be vacant, or existing households move from small dwellings into larger, vacant ones. Either change increases the amount of housing services that is consumed. The price of housing services also increases in the short run to bring realized demand into equilibrium with available supply. The short-run price elasticity of supply is the ratio of relative changes in the occupancy rate to relative changes in price.

The short-run supply of housing services is very inelastic, but not completely so. We estimate that in a tight market (one with a 96 percent initial rental occupancy rate), the short-run price elasticity of

the supply of housing services is 0.24; and that in a loose market (one with a 90 percent initial rental occupancy rate), the short-run elasticity is 0.83. The short-run elasticity varies with market condition because the lower the initial occupancy rate, the more the occupancy rate can expand and absorb part of a demand increase (by definition, the occupancy rate cannot exceed 100 percent).

In the intermediate run, repairs of existing housing and additions to the housing inventory gradually build toward their long-run levels, and the price increase necessary to clear the market gradually falls to its long-run level. Consequently, the price elasticity of supply gradually rises from its short- to its long-run value. For a tight market, we estimate the supply elasticity as 0.49 one year after a demand increase begins, and as 2.35 three years after. Corresponding estimates for a loose market are 1.25 and 2.41.

The price elasticity of supply, together with that of demand, determines whether demand increases cause large or small increases in the price of housing services. The supply elasticity estimates in this report help explain why the experimental housing allowance program that was part of Rand's Housing Assistance Supply Experiment caused only small price increases. Even in the short run, occupancy changes prevent the price elasticity of housing supply from dropping to zero. Consequently, even in the early years of the housing allowance program, supply responses moderated program-induced price increases.

When the occupancy rate expands to absorb part of any excess demand in a housing market, it not only reduces the price increase caused by excess demand, but it also makes the occupancy rate an indicator of market conditions. This study estimates the following relationships between excess demand, rental vacancy rate (1.0 less the occupancy rate), and the degree to which price exceeds its long-run equilibrium level:

Excess Demand (%)	Rental Vacancy Rate (%)	Extra Price (%)
8	3.7	9.7
4	5.0	4.4
0	6.6	0.0
-4	8.7	-3.6
-8	11.1	-6.5

Of the three indicators of housing-market condition, only vacancy rate is routinely measured. The results of the study therefore enable observed vacancy rates to be translated into either or both of the other two indicators.

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I. INTRODUCTION

This report analyzes the price elasticity of the supply of rental housing services, defined as the percentage increase in supply associated with a one percent increase in price. Although our definition emphasizes supply as a function of price, the purpose of the analysis is the opposite: to predict the price changes associated with supply responses to shifts in demand.

The analysis abstracts the multidimensional nature of housing into one homogeneous good. Large and high-quality dwellings differ from small and poor-quality ones only in the amount of housing services they provide. Under that concept, the classic microeconomic constructs of demand and supply curves are applicable to housing markets.[1]

We focus on the flow of housing services as opposed to stocks of housing capital. "Housing quantity" is the amount of housing services produced by both capital inputs (land and structures) and energy and other current inputs during a period. "Housing price" is the rent per unit of housing services during the period.

When the demand for rental housing services increases, the market responds with a combination of supply increases (to accommodate the new demand) and price increases (to bring realized demand into equilibrium with current supply).[2] The demand increase causes the commodity to

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[1] The concept was originally developed by Muth (1960). Its implications have been explored by Olsen (1969).

[2] In other words, the demand curve shifts to the right, and increased supply accommodates some of the shift. Then increased price moves consumers upward along the demand curve until housing consumption equals supply.

become scarce; consumers react by bidding up its price. Suppliers then find it profitable to increase output, causing price to be bid down again. The balance changes over time between the supply and the price responses, with the role of supply steadily increasing.

At any stage in the supply/demand cycle, the size of the price increase depends on three factors: (a) the size of the demand shift, (b) the responsiveness of supply to price, and (c) the responsiveness of demand to price. We measure the demand shift by the percentage increase in demand that would occur if price remained constant. We measure supply responsiveness by the price elasticity of supply, which is the percentage increase in the supply of housing services resulting from a one percent increase in price. Analogously, we measure demand responsiveness by the price elasticity of demand, which is the percentage decrease in the demand for housing services resulting from a one percent increase in price.

The three factors have the following fundamental relation to price:

$$\text{Percentage price increase} = \frac{\text{Percentage demand shift}}{\text{Price elasticity of supply} + \text{Price elasticity of demand}} . \qquad (1.1)$$

Expressed in relative-change form, the demand curve is $q = d - Sp$ and the supply curve is $q = Yp$, where q = relative change in quantity, p = relative change in price, d = relative shift in demand, S = price elasticity of demand, and Y = price elasticity of supply. The intersection of the demand and supply curves establishes $p = d/[Y + S]$, the algebraic form of Eq. (1.1).

The present analysis assumes that the price elasticity of demand for housing services is 0.5. Mayo's (1981) literature review found estimates of the price elasticity of rental demand varying from 0.17 to 1.28. However, the estimates clustered around a central tendency of 0.5. Two studies that found a value of 0.5 are Straszheim's (1973) analysis of data from a San Francisco Bay area transportation study (the exact estimate was 0.53) and Vaughn's (1976) analysis of U.S. Census data (the exact estimate was 0.48). Those middle-of-the-decade estimates are smaller than that prevailing at the start of the decade: DeLeeuw's (1971) estimate of 0.71. And they are larger than the end-of-the-decade finding of the Housing Allowance Demand Experiment: Friedman and Weinberg's (1978) estimate of 0.22 for low-income households. However, DeLeeuw's estimate was considered by later studies to be slightly too high; and Friedman and Weinberg conclude that the price elasticity of demand varies with income and warn that their estimate is therefore too low to be used for general market studies.

Using 0.5 for the price elasticity of demand in our fundamental equation, we find that if the price elasticity of supply is zero, then the percentage price increase equals twice the percentage demand shift. On the other hand, if the price elasticity of supply is infinitely large, demand shifts cause no price increases, no matter what the price elasticity of demand.

When analyzing housing-market behavior, it is often convenient to assume that in the short run the supply elasticity is zero, and that in the long run it is infinitely large. Those assumptions lead to the conclusion that the price increases caused by a demand shift will be large in the short run (double the percentage demand shift) but very small in the long run (ultimately zero, when supply has responded completely to the demand shift).

More precise analysis must recognize that the supply elasticity is never vanishingly small; nor is it ever infinitely large. Rather, it lies between the two extremes, its size depending on time since the demand shift began, and market condition at the start of the demand shift.

To explain why supply elasticity varies with time and market conditions, we examine the components of supply response to demand shifts. The supply of housing services available to consumers can increase in three ways: (a) existing housing can be upgraded by repairs; (b) the housing inventory can be expanded either by using existing residential land more intensely or by increasing the amount of residential land; and (c) the proportion of existing housing that is occupied can be increased. The overall supply elasticity is a composite of all three components. The composite elasticity increases with time since the demand shift began because of the time suppliers need to repair existing housing and add new housing. It is larger in loose markets (ones with relatively low occupancy rates) than in tight ones because markets with more vacancies can accommodate more demand.

Section II of this report analyzes each component of supply response separately. It presents price elasticities for the repair, inventory, and occupancy responses to demand shifts. It reviews the literature on all three and offers new estimates for the second and third (the estimates are based on the analysis of Annual Housing Survey data from the U.S. Census Bureau reported in Appendix B).

Section III combines the three individual supply elasticities into a composite elasticity. It accomplishes the integration using a model of housing-market responses to demand shifts presented in Appendix C. The model was built during the Housing Assistance Supply Experiment to explain the housing market's response to demand shifts caused by an experimental housing allowance program.

II. COMPONENT SUPPLY ELASTICITIES

The supply of housing services can be altered in three different ways. First, existing housing can be upgraded by making repairs or can be allowed to deteriorate by withholding repairs. Second, the housing inventory can be changed by developing existing residential land more or less intensively, or by adding to or subtracting from the amount of land available for residential use. Third, the existing inventory can be used more or less intensively according to changes in the occupancy rate. The components of supply change are fundamentally different, so it is not surprising that their price elasticities differ greatly.

This analysis finds that the repair elasticity is 0.2, that the inventory elasticity due to input substitution is 6.7, that the total inventory elasticity is 11.3, and that the occupancy elasticity ranges from 0.3 in a tight market (with a 96 percent initial rental occupancy rate) to 0.9 in a loose market (with a 90 percent initial rental occupancy rate). Table 1 compares those estimates with others found in the literature on housing. Not only is there considerable disagreement, even with four categories established to prevent improper comparisons, but also there are not enough studies in each category to permit central tendencies to emerge (as they do in studies of the price elasticity of demand). Nevertheless, the bulk of the studies firmly support the conclusions that the repair elasticity is very low, that the inventory elasticity is very large, and that the occupancy elasticity is meaningfully greater than zero.

Table 1

LITERATURE ESTIMATES OF COMPONENT SUPPLY ELASTICITIES

Source	Estimate
Repair Elasticity	
Ozanne and Struyk (1976, p. 22)	0.2 to 0.3
Rydell and Neels (1982)	0.2
Inventory Elasticity[a]	
Muth (1969, p. 55)	20.0
Rydell (Appendix A, this report)	6.7
Inventory Elasticity[b]	
DeLeeuw and Ekanem (1971, p. 814)	0.3 to 0.7
DeLeeuw and Struyk (1975, p. 15)	∞
Ozanne and Thibodeau (1980, p. 31)	∞
Rydell (Appendix B, this report)	11.3
Occupancy Elasticity	
DeLeeuw and Ekanem (1971, p. 812)	0
Rydell (1979, p. 6)	3.4
Rydell (Appendix B, this report)[c]	$0.5[F^{-10} - 1]$

[a]Due only to input substitution.

[b]Total.

[c]The responsiveness of occupancy to price depends on the initial rental occupancy rate (F). For example, where F = 0.96 (a tight market), the occupancy elasticity is 0.25, and where F = 0.90 (a loose market), the occupancy elasticity is 0.93.

REPAIR ELASTICITY

Owners of existing housing choose a repair policy that makes the marginal repair dollar purchase a dollar's worth of housing services. If there is a rise in the market price at which housing services can be sold, then increasing repairs is an economically rational step. On the other hand, if the price of housing services falls, then the economically rational course is to reduce repairs and allow deterioration to progress.

The actual relationship between the economically optimal level of repairs and the price of housing services depends on the technical relationship between repair inputs and housing service output. Rydell and Neels (1982), using panel data on rental housing in Brown County, Wisconsin, and St. Joseph County, Indiana, to analyze that relationship, find the repair elasticity to be 0.2. Their estimate agrees closely with the 0.2 to 0.3 found by Ozanne and Struyk (1976), who used panel data on renter- and owner-occupied housing in Boston.

Considering how difficult it is to significantly alter housing once it is built, it is not surprising that the repair elasticity is very low. A low repair elasticity does not however mean that the overall supply of housing services is extremely inelastic (in the long run, inventory change will accommodate demand shifts with only small price changes; in the short run, occupancy changes will accommodate part of a demand shift and reduce the price changes necessary to clear the market). Rather, a low elasticity means that repairs will be a minor part of a housing market's aggregate response to demand shifts.

INVENTORY ELASTICITY

The housing inventory can be increased either by developing existing residential land more intensely or by bidding land away from nonresidential uses. The price elasticity of the inventory therefore depends both on the degree to which nonland inputs can be substituted for land inputs to the production of housing services, and on the price elasticity of residential land. The larger the input substitution elasticity and the larger the land elasticity, the larger the inventory elasticity.

In 1964, Muth set out a theory of the relationship between input substitution elasticity, land elasticity, and inventory elasticity. He used that theory to estimate 20 as the inventory elasticity caused by input substitution (Muth, 1969). However, his estimate did not do justice to his theory, since in 1969 only crude estimates of input substitution elasticities were available. More recently, Neels (1981) constructed excellent estimates of housing input substitution elasticities using Housing Assistance Supply Experiment data. Applying Neels's input substitution elasticities to Muth's theory, in Appendix A we find 6.7 to be the inventory elasticity due to input substitution.

We conclude that the inventory elasticity is at least as large as 6.7, because that much is caused by input substitution alone. Appendix B analyzes the variation in the price of housing services across metropolitan areas of different sizes, using Annual Housing Survey data from the U.S. Census Bureau, and concludes that the total inventory elasticity is 11.3. Subtracting 6.7 from 11.3, we then estimate that the ability to bid land away from nonresidential uses contributes 4.6 to the total inventory elasticity.

Confirming our inventory elasticity estimate in the housing literature is difficult because there are few relevant studies. Many studies of "supply elasticity" are concerned with housing capital, not housing services. They estimate the responsiveness of housing capital (land plus improvements) to the sales price per unit of capital, rather than the responsiveness of the flow of housing services to the rent per unit of those services.[1]

The few studies focused on the elasticity of the inventory of housing services show extreme variation in their estimates. DeLeeuw and Ekanem (1971, p. 814) conclude that the inventory elasticity is between 0.3 and 0.7. But four years later, when building the Urban Institute's housing-market simulation model, DeLeeuw judges the elasticity to be infinitely large (DeLeeuw and Struyk, 1975, p. 15).

Ozanne and Thibodeau (1980) suggest that the ambitious econometrics used in the DeLeeuw and Ekanem study required better data than was available at the time. However, their own attempt to estimate the inventory elasticity also pressed available data (Ozanne and Thibodeau, 1980, p. 31). They attempted to simultaneously estimate the price elasticity of housing supply and the price elasticity of residential land. Since changes in the price of housing services and in the price of land are highly correlated, it is not surprising that the precision of estimation suffered. The result was that they could not find the inverse of the inventory elasticity to be significantly different from zero, and hence concluded that the inventory elasticity was infinitely large.

OCCUPANCY ELASTICITY

Ultimately, repair and inventory changes are the market's total response to demand shifts. However, the responses take time. In the meantime, realized demand must be brought into equilibrium with current

[1] Estimates of the price elasticity of housing capital rather than the price elasticity of housing services are found for example in Smith (1976, p. 402), Bradbury et al. (1977, p. 72), and Follain (1979, p. 197).

supply, either by changes in the occupancy rate or by changes in the price of housing services.

The reason occupancy rates change is that demand pressure shortens the time a dwelling remains vacant after one occupant moves out and before another moves in. Consequently, on any given day the proportion of dwellings occupied is greater than it would be without the increased demand. But because the occupancy rate can never be larger than 100 percent by definition, the closer the rate is to that limit the less it responds to demand pressure.

Although the importance of variations in occupancy rate has been clear for almost thirty years--having been pointed out in Rapkin, Winnick, and Blank's (1953) classic study of housing-market behavior and underlined in DeLeeuw and Ekanem's (1973) analysis of market dynamics--the amount of a demand shift absorbed by the occupancy rate, rather than by changes in price, has proved difficult to quantify. For example, DeLeeuw and Ekanem (1971) could not find a statistically significant relationship between the price of housing services and the occupancy rate. Consequently, many housing-market models implicitly assume that the price elasticity of the occupancy rate is zero, and therefore overestimate the short-run price increases caused by demand shifts. Examples are the housing-market simulation models of the Urban Institute (DeLeeuw and Struyk, 1975) and of the National Bureau of Economic Research (Kain and Apgar, 1977). Then, at the end of the 1970's, Housing Assistance Supply Experiment data for two metropolitan areas suggested 3.4 as the price elasticity of the occupancy rate (Rydell, 1979).

Because of the small sample size the present study was undertaken in hopes of obtaining a more reliable estimate.

The estimate in the present report uses data from 59 metropolitan areas; it also draws on the extensive hedonic index analyses of rent done by the Urban Institute (see for example Follain and Malpezzi, 1980). Such work ultimately separated price variation from quantity variation across the entire sample of 59 metropolitan areas surveyed in the Annual Housing Survey. Never before have such excellent price indexes been available for so large a sample.[2]

Regression analysis of the relation between the price of rental housing services and the rental occupancy rate (see Appendix B) shows that the price elasticity of the occupancy rate equals $0.5\ [F^{-10} - 1]$, where F is the initial rental occupancy rate. Using that formula, we can compute the occupancy elasticity for any market condition--for example, those given in Table 2. As anticipated, the occupancy elasticity is higher in loose markets than in tight ones. For markets in long-run equilibrium (where we estimate that the occupancy rate is 0.934; see Appendix B), the occupancy elasticity is 0.5.

The occupancy elasticities given in Table 2 are precisely correct only for small demand shifts. There are three ways to analyze large demand shifts: (a) accept the approximation inherent in using Table 2; (b) compute the occupancy response in small steps, revising the occupancy elasticity as each intermediate occupancy rate is established; or (c) use Table 3.

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[2] The author is indebted to Larry Ozanne of the Urban Institute for providing the area-by-area price of rental housing services listed in Appendix B.

Table 2

EFFECT OF MARKET CONDITION
ON OCCUPANCY ELASTICITY

Initial Rental Occupancy Rate	Price Elasticity of Occupancy Rate
.98	.11
.97	.18
.96	.25
.95	.34
.94	.43
.93	.53
.92	.65
.91	.78
.90	.93
.89	1.10
.88	1.30

SOURCE: Equation (B.12), Appendix B.

The first column in Table 3 gives excess demand, which is the percentage difference between the current level of demand and the level that would be in long-run equilibrium with the existing housing inventory. The second and third columns give the corresponding occupancy rate and the ratio of current housing price to the long-run equilibrium price.

With zero excess demand, occupancy rate and price are at their long-run equilibrium levels. As excess demand increases, both occupancy rate and price increase. Dividing the percentage changes in occupancy rate by the percentage change in price, as excess demand changes, yields the occupancy elasticity.

Table 3

EFFECT OF EXCESS DEMAND ON OCCUPANCY RATE AND PRICE OF HOUSING SERVICES

Excess Demand[a] (%)	Rental Occupancy Rate	Price of Housing Services[b]
20	.985	1.294
18	.983	1.257
16	.980	1.222
14	.977	1.188
12	.973	1.156
10	.968	1.125
8	.963	1.097
6	.957	1.070
4	.950	1.044
2	.943	1.021
0	.934	1.000
- 2	.924	.981
- 4	.913	.964
- 6	.902	.948
- 8	.889	.935
-10	.875	.923
-12	.860	.913
-14	.844	.905
-16	.828	.898
-18	.811	.892
-20	.793	.887

SOURCE: Equations (B.13) and (B.14), Appendix B.

[a]Percentage difference between the current level of demand for housing services and the level that would be in long-run equilibrium with the current inventory of housing services.

[b]Ratio of price to the long-run equilibrium price corresponding to the current housing inventory.

For example, suppose a 10 percent increase in demand occurs in a market where the initial rental occupancy rate is 0.95. The excess demand corresponding to the initial occupancy rate is 4 percent. The demand increase raises excess demand to 14 percent (assuming there has not yet been time for repair and inventory responses). As a result, the occupancy rate increases by 2.8 percent, from 0.950 to 0.977, and price increases by 13.8 percent, from 1.044 to 1.188. The ratio of those percentage changes, 0.20, is the occupancy elasticity. It is considerably different from the 0.34 occupancy elasticity shown in Table 2, corresponding to an initial occupancy rate of 0.95, because the demand shift was large. Note that the 0.20 elasticity is roughly the average of the elasticities shown in Table 2 for the range (0.95 to 0.98) through which the occupancy rate passes as it adjusts to the 10 percent demand increase.

III. COMPOSITE SUPPLY ELASTICITIES

In the short run, before the repair and inventory responses to a demand increase have begun, the composite supply elasticity is essentially the occupancy elasticity--0.25 in a tight market and 0.93 in a loose market. In the long run, when the repair and inventory responses constitute the market's complete response to a demand shift, the composite supply elasticity equals the sum of the repair and inventory elasticities--11.5.

In the meantime (the intermediate run), while the repair and inventory responses to a demand shift are going on, the composite supply elasticity gradually increases from its short-run to its long-run value. It can be thought of as a weighted average of the occupancy, repair, and inventory elasticities, with the weight of the first decreasing over time.

The market behavior model presented in Appendix C demonstrates how the composite supply elasticity changes after a demand increase starts. It traces the supply and price increases in the demand for housing services. The ratio of relative increase in supply to relative increase in price is the composite supply elasticity.

An instantaneous increase in demand gives an unrestricted view of supply response, although actual demand increases rarely occur instantaneously. For example, the housing allowance program tested by the Housing Assistance Supply Experiment took three to four years to reach its equilibrium level. When analyzing the price increases caused by that program, we took the phased demand increase into account (see

Rydell, Neels, and Barnett, 1982). However, for the present analysis we did not want the effect of lags in the supply response to be obscured, and therefore telescoped lags in the demand increase into an instantaneous effect.

MARKET EFFECTS OF DEMAND INCREASE

When the aggregate demand for housing services in a rental market increases, the price per unit of that service is bid up enough to clear the market. However, suppliers of housing services soon notice the price increase and find it profitable to expand supply. As supply expands, prices are bid downward, and consumption increases until the market clears again.

Supply responds to demand shifts in three ways. First, some existing rental dwellings are upgraded to take advantage of the higher prices. Second, the inventory of rental housing changes as the result of new construction, demolition, or conversion; the last includes dwellings that shift between the rental and the ownership markets. Third, the occupancy rate within the rental inventory rises or falls to accommodate greater or lesser consumption.

Our model of market behavior estimates the time-path of each supply response separately, then sums them. The results for first a tight, then a loose market are plotted in Figs. 1 and 2.[1] The pattern is

[1] The choice of a 3 percent demand increase does not affect our elasticity results, because both the supply and the price increases are proportional to the size of the demand increase. The tight market in our analysis has an initial rental occupancy rate of 96 percent; the loose market has an initial rate of 90 percent. Supply and price increases in alternate situations can readily be estimated using the model presented in Appendix C.

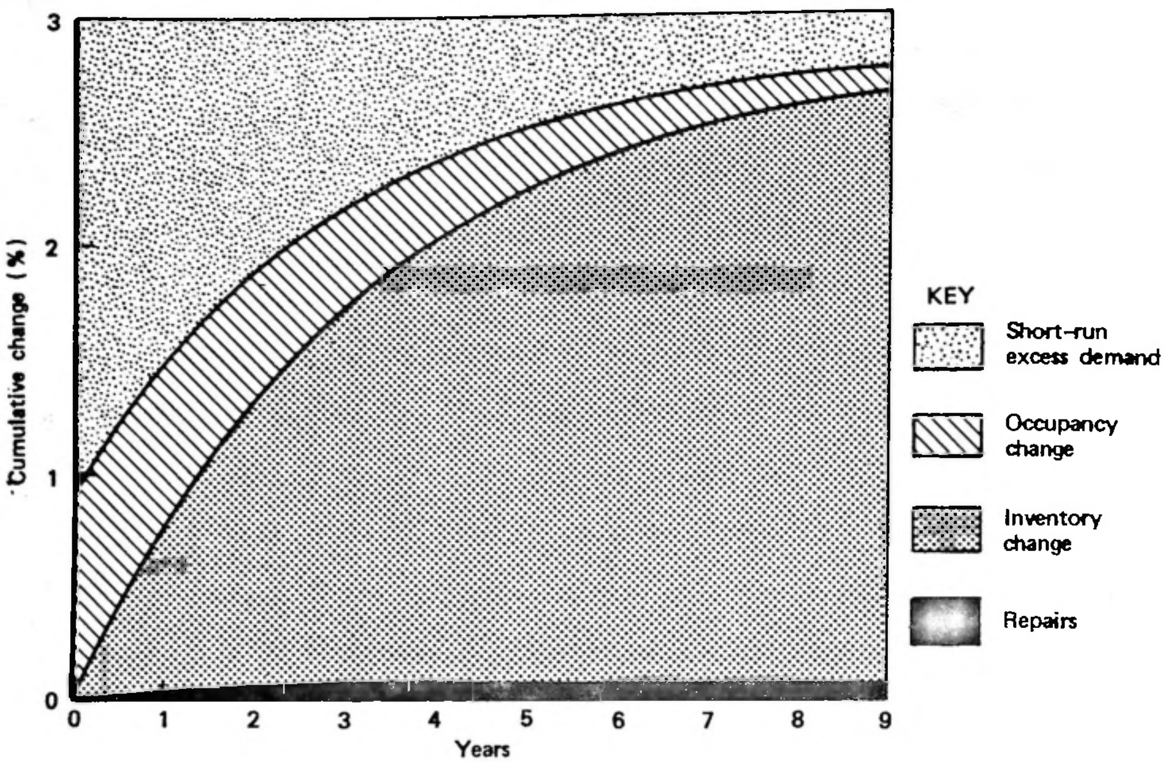

Fig. 1 — Supply response to instantaneous 3 percent demand increase (tight market)

similar in both markets. The repair response and inventory change occur steadily but slowly, each year accommodating more of the demand shift but taking many years to eliminate the difference between the desired amount of housing services and the current amount. The occupancy adjustment accommodates between one-third and two-thirds of the demand change left after accounting for the repair and inventory responses.

The occupancy change is smaller in the tight market than in the loose one. Since occupancy rates can never by definition exceed 100

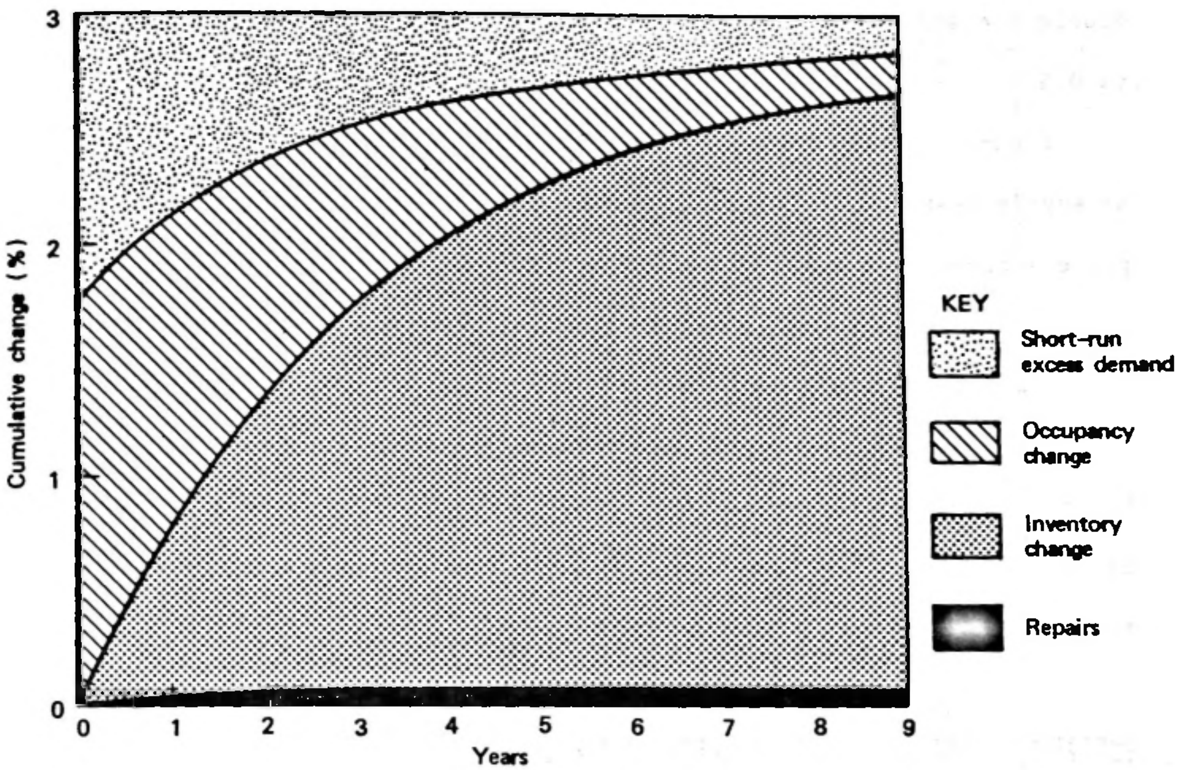

Fig. 2 — Supply response to instantaneous 3 percent demand increase
(loose market)

percent, the higher they are initially, the less increased demand they can absorb.

In our model, the supply response lags behind the demand increase. Consequently, to clear the market at any time requires price changes. The price of rental housing services must rise to eliminate the short-run excess demand in Figs. 1 and 2.[2] The required price increases are

[2] Short-run excess demand is the demand increase less the repair, inventory, and occupancy responses. Note that in estimating short-run excess demand, we use the demand that would exist at the long-run equilibrium price of housing services. Of course, once price adjusts to clear the market, short-run excess demand becomes zero.

double the short-run excess demand, since the price elasticity of demand is 0.5.

Figure 3 plots the time-path of the price increase. If there was no supply response, a 3 percent demand increase would cause a 6 percent price increase. However, in the short run, occupancy change limits the price increase to two-thirds the potential amount in the tight market and to one-third the amount in the loose market. In the intermediate and long runs, inventory changes (and to a small extent repairs of existing housing) dramatically reduce the price increase. The price increase is always higher in the tight market because the occupancy response is smaller there than in the loose market.

IMPLIED COMPOSITE SUPPLY ELASTICITIES

The ratio of the percentage change in supply to the percentage change in price is by definition the price elasticity of supply. Table 4 divides the total supply increases shown in Figs. 1 and 2 by the price increases illustrated in Fig. 3 to reveal the implied composite supply elasticities. It shows that the composite supply elasticity is always greater than zero, even in the short run, and that it increases rapidly after the demand increase begins. One year after the start of the demand increase, it is double its short-run level in the tight market and 50 percent greater than its early level in the loose market. However, even nine years after the start of the demand increase, it is only half the long-run level in the tight market and only two-thirds that level in the loose market.

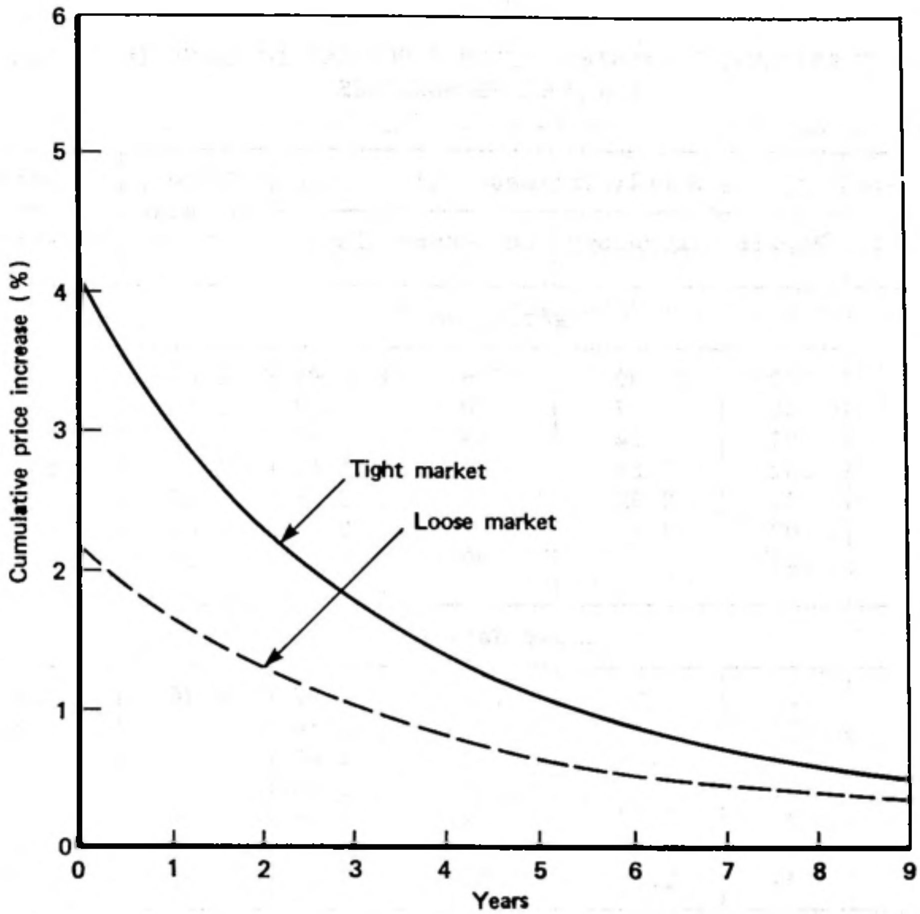

Fig. 3 — Price increases caused by instantaneous 3 percent demand increase

The short-run composite supply elasticity (0.24 in the tight market and 0.83 in the loose market) is almost equal to the occupancy elasticity (0.25 in the tight market and 0.93 in the loose market).[3] The long-run composite supply elasticity equals the sum of the repair and inventory elasticities (11.5 in both markets).

[3] The short-run composite supply elasticity is slightly less than the occupancy elasticity because occupancy responds to the gap between current price and long-run equilibrium price, rather than to the gap between current price and initial price [see Eq. (C.14), Appendix C].

Table 4

MARKET RESPONSE TO INSTANTANEOUS 3 PERCENT INCREASE IN DEMAND FOR HOUSING SERVICES

Years Since Demand Increased	Supply Increase (%)				Price Increase (%)	Composite Supply Elasticity
	Repair	Inventory	Occupancy	Total		
Tight Market[a]						
0	.00	.00	.96	.96	4.07	.24
1	.00	.77	.70	1.48	3.04	.49
3	.01	1.74	.38	2.13	1.74	1.22
5	.02	2.25	.20	2.47	1.05	2.35
7	.02	2.52	.11	2.66	.69	3.86
9	.03	2.67	.06	2.75	.49	5.58
∞	.05	2.83	.00	2.87	.25	11.50
Loose Market[b]						
0	.00	.00	1.87	1.87	2.26	.83
1	.00	.77	1.37	2.14	1.71	1.25
3	.01	1.74	.73	2.48	1.03	2.41
5	.02	2.25	.39	2.66	.67	3.97
7	.02	2.52	.21	2.76	.48	5.75
9	.03	2.67	.12	2.81	.38	7.44
∞	.05	2.83	.00	2.87	.25	11.50

SOURCE: Equations (C.17) - (C.20), Appendix C.
NOTE: 0 years = short run; ∞ years = long run.

[a] Initial rental occupancy rate = 0.96.

[b] Initial rental occupancy rate = 0.90.

IV. CONCLUSIONS

The price elasticity of the supply of housing services determines the price increase necessary to accommodate an increase in demand.[1] A high supply elasticity implies that only a small price increase will be necessary. Conversely, a low supply elasticity implies a large price increase. The specific relationship between price increase and supply elasticity is as follows:

$$\text{Price increase (\%)} = \frac{\text{Demand increase (\%)}}{\text{Supply elasticity} + 0.5} . \qquad (4.1)$$

However, the relationship between demand increase and price increase is not as simple as the formula implies, because supply elasticity is not a constant, but rather varies with both market condition and time since a demand increase begins.

In the short run, occupancy change is the only possible supply response to increased demand. The amount of the response varies with market condition: it is smaller in a tight market (high initial occupancy rate) than in a loose one (low initial occupancy rate).

In the intermediate run, repairs to existing housing accommodate a small part of the demand increase, and expansion of the housing inventory ultimately accommodates almost all of it. But those supply responses occur over a period of several years. So in the intermediate

[1] For clarity of exposition, our conclusions are stated in terms of increases--price increases caused by demand increases. However, they also apply to decreases--price decreases caused by demand decreases.

run, the responsiveness of supply to demand varies with elapsed time since the demand increase began. Moreover, while the repair and inventory responses are "chasing" the demand increase, occupancy change continues to play a part in accommodating increased demand (albeit a diminishing one). Hence, the intermediate-run supply response varies with market condition as well as with time.

In the long run, the repair and inventory responses completely adjust supply to the new level of demand, and occupancy change is no longer needed. In other words, the market reaches a new long-run equilibrium between supply and demand, reestablishing the normal occupancy rate.

Table 5 summarizes our estimates of supply elasticities and the consequent price increases caused by demand increases. For simplicity, the table presents estimates for only two market conditions (a tight market with a 96 percent initial rental occupancy rate, and a loose market with a 90 percent initial rental occupancy rate), and for only seven time intervals.[2]

Our results show that the supply elasticity increases with elapsed time from 0.24 to 11.50 in a tight market and from 0.83 to 11.50 in a loose market. The corresponding price increases per one percent increase in demand start at 1.35 in a tight market and 0.75 percent in a loose market, and decrease over time to 0.08 percent in both.

Our supply elasticity results enable us to answer a fundamental question posed by the Housing Assistance Supply Experiment: How much do

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[2] Appendix C gives a method of estimating the supply elasticities for any market condition at any time.

Table 5

COMPOSITE SUPPLY ELASTICITY

Years Since Demand Increased	Composite Supply Elasticity	Percentage Price Increase[a]
	Tight Market[b]	
0	.24	1.35
1	.49	1.01
3	1.22	.58
5	2.35	.35
7	3.86	.23
9	5.58	.16
∞	11.50	.08
	Loose Market[c]	
0	.83	.75
1	1.25	.57
3	2.41	.34
5	3.97	.22
7	5.75	.16
9	7.44	.13
∞	11.50	.08

SOURCE: Table 4 and Eq. (4.1).
NOTE: 0 years = short run; ∞ years = long run.

[a] Per one percent demand increase.

[b] Initial rental occupancy rate = 0.96.

[c] Initial rental occupancy rate = 0.90.

supply responses to program-induced demand limit program-induced increases in the price of housing services? The housing allowance program caused the demand for rental housing services in the recipient submarket to increase by 4.6 percent in Brown County, Wisconsin, and by 5.6 percent in St. Joseph County, Indiana.[3] If there had been no supply

[3] Allowance program regulations prohibited recipients from living in substandard housing, and their low incomes excluded them from luxury housing; so program-induced demand focused on the middle of the market.

response to the demand increases, the price of rental housing services would have risen by double the demand shift--9.2 percent in Brown County, and 4.2 percent in St. Joseph County. However, using the price elasticities of supply in Table 5, we find that supply response reduces the potential price increase by one-third to one-half in the short run (see Table 6, "Supply Response Only" column), and by considerably more in the intermediate and long runs.[4]

We estimated the effect of supply response on the price increases caused by demand shifts by assuming an instantaneous demand change (see Sec. III). However, the increased demand generated by the housing allowance program built up gradually over three to four years as the program grew to its equilibrium level. Consequently, before the full demand shift had occurred, the repair and inventory responses had already started. As a result, the price increases caused by the allowance program were only about half as large as they would have been had the program reached its equilibrium level immediately (see the last column in Table 6).

The role played by occupancy rate in the housing-market response to demand change illustrates a familiar but not well understood measure of market condition. All observers of housing-market behavior know that high occupancy rates--i.e., low vacancy rates--indicate excess demand and high prices. But how much excess demand and how high a price?

Table 7 provides the answer. The first column lists the percentage of excess demand--the amount of a demand increase not yet accommodated by repair and inventory responses. The second column lists the rental

[4] The rental occupancy rates were 96 percent in Brown County and 90 percent in St. Joseph County before the start of the allowance program.

Table 6

MAXIMUM PRICE INCREASES CAUSED BY HOUSING ALLOWANCE PROGRAM

Location	Demand Increase[a] (%)	Maximum Price Increase (%)		
		No Supply Response[b]	Supply Response Only[c]	Supply Response and Lagged Demand Increase[d]
Brown County, Wisconsin	4.6	9.2	6.2	3.5
St. Joseph County, Indiana	5.6	11.2	4.2	2.4

SOURCE: Rydell, Neels, and Barnett (1982), and Table 5.

[a]Increase in the demand for rental housing services caused by the housing allowance program in the submarket patronized by allowance recipients (two-thirds of the Brown County rental market, one-half of the St. Joseph County rental market).

[b]If there are no supply responses, the percentage price increase needed to clear the market is double the percentage demand increase (because the price elasticity of demand is 0.5).

[c]Short-run price increase caused by an instantaneous demand increase, estimated using results given in Table 4.

[d]Maximum price increase caused by the housing allowance program, between two and three years after the program started. Before and after that time, the program-induced price increases were considerably smaller; see Rydell, Neels, and Barnett (1982).

vacancy rates (100 minus the occupancy rate) corresponding to each excess demand. The final column lists the difference between the current price of housing services and the long-run equilibrium price (the "extra price").

Zero excess demand means that demand is in long-run equilibrium with supply; producers of housing service have no incentive to either expand or contract the housing inventory (although the normal process of decay and replacement continues). The vacancy rate is 6.6 percent--the

Table 7

EFFECT OF EXCESS DEMAND ON RENTAL VACANCY RATE AND PRICE OF HOUSING SERVICES

Excess Demand (%)	Rental Vacancy Rate (%)	Extra Price (%)
20	1.5	29.4
18	1.7	25.7
16	2.0	22.2
14	2.3	18.8
12	2.7	15.6
10	3.2	12.5
8	3.7	9.7
6	4.3	7.0
4	5.0	4.4
2	4.7	2.1
0	6.6	.0
-2	7.6	-1.9
-4	8.7	-3.6
-6	9.8	-5.2
-8	11.1	-6.5
-10	12.5	-7.7
-12	14.0	-8.7
-14	15.6	-9.5
-16	17.2	-10.2
-18	18.9	-10.8
-20	20.7	-11.3

SOURCE: Table 3.

[a]Percentage difference between the current level of demand for housing services and the level that would be in long-run equilibrium with the current inventory.

[b]Vacant dwellings for rent, as a percent of occupied rental dwellings plus vacant dwellings for rent (U.S. Census definition of the "rental vacancy rate").

[c]Percentage deviation of the current price of housing services from the long-run equilibrium price.

long-term average rental vacancy rate in the United States--and price is at its long-run equilibrium level.

With excess demand, vacancy rates fall and price rises. For example, the table shows that a 4 percent increase in demand (net of any repair and inventory responses) causes the vacancy rate to fall to 5 percent and increases the price of housing services by 4.4 percent. Consequently, if we encounter a housing market with a 5 percent rental vacancy rate, we know there is 4 percent excess demand and that the price of housing services is 4.4 percent above its long-run equilibrium level.

Two cautions must be heeded regarding Table 7. First, when applying its results elsewhere, the rental vacancy rate as defined and measured by the U.S. Census Bureau must be used. Other measures of vacancy rate would be inappropriate because the empirical work behind the table on the relation of vacancy rates to excess demand and prices used the Census Bureau's rate. Second, the vacancy rates in Table 7 are strictly correct only for markets with an average incidence of renter mobility (about 35 percent of renter households moving during a 12-month period). A higher mobility rate increases the vacancy rate that corresponds to a given excess demand; a lower mobility rate lowers the vacancy rate for any excess demand (see Appendix D).

Table 7 highlights the asymmetrical effect of excess demand on occupancy and price. A demand surplus of 20 percent reduces the vacancy rate by only 5.1 percentage points, to 1.5 percent, while raising the price by 29.4 percent. In contrast, a demand deficiency of 20 percent increases the vacancy rate by 14.1 percentage points while decreasing price by only 11.3 percent. The asymmetry occurs because occupancy rates cannot respond as well to demand pressures in tight as opposed to loose markets; rather, price change must do more to clear the market. In other words, the pattern in Table 7 reflects the fact that the price elasticity of the occupancy rate is lower in tight markets than in loose ones.

The pattern suggests an interesting check on our results. Assuming that excess demand is approximately normally distributed (i.e., has a symmetrical, bell-shaped distribution) across housing markets at any time, the asymmetrical effect of excess demand on vacancy rates should cause the distribution of vacancy rates to be skewed toward high rates. That in fact is the case. For example, only 3 of the 59 metropolitan areas selected for the Annual Housing Survey have vacancy rates below 4 percent, while 15 out of the 59 have vacancy rates above 10 percent.

The asymmetry also points out an irony in using vacancy rates to measure housing-market condition. In conditions of surplus demand where there is great concern about a housing crisis, small changes in vacancy rates indicate relatively large changes in excess demand and price. Consequently, the only available operational measure of market condition is least precise where it is most used.

Appendix A

INVENTORY ELASTICITY ESTIMATED FROM INPUT SUBSTITUTIONS

The flow of housing services comes from land, improvements, energy, and other inputs.[1] The flow price is realized as rent per unit of housing service. The price elasticity of housing supply is the percentage change in supply per one percent increase in price.

The inventory of housing services can be increased either by substituting nonland inputs for the land input or by varying the land input. Muth's (1964) theory shows how much of each supply elasticity is due to each cause. Assuming that the price of nonland inputs is not affected by the scale of housing production, Muth's theory concludes that

$$Y = \frac{K\sigma_k + E\sigma_e + T\sigma_t}{L} + \frac{\varepsilon}{L}, \qquad \text{(A.1)}$$

where Y = price elasticity of the inventory of housing services,

ε = price elasticity of land,

σ_i = elasticity of substitution between nonland input i and land,

L = share of gross rent due to land,

K = share of gross rent due to improvements,

E = share of gross rent due to energy,

T = share of gross rent due to other inputs.

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[1] Energy inputs include electricity, gas, fuel oil, and coal used for lighting, heating, and cooking. Other inputs include water and sewer services, management and janitorial services, and landlord labor.

The first term gives the component of supply elasticity due to input substitution; the second gives the component due to varying land. The elasticity increases with increases in both input substitutability and price elasticity of land.

Table A.1 derives estimates for Eq. (A.1) using Housing Assistance Supply Experiment data. Substituting the parameters into the first term of Eq. (A.1) yields 6.7 as the inventory elasticity due to input substitution. By subtraction (anticipating the conclusion in Appendix B that the total inventory elasticity is 11.3), we then estimate 4.6 as the inventory elasticity due to variations in the amount of residential land.

Table A.1

PARAMETERS FOR INVENTORY ELASTICITY FORMULA

Input to Production of Housing Supply	Share of Gross Rent	Elasticity of Substitution Between Nonland and Land Inputs
Land	.09	(a)
Improvements	.55	.60
Energy	.21	.61
Other current	.15	.94

SOURCE: Calculated from Housing Assistance Supply Experiment data. Input shares estimated in Table A.2; substitution elasticities estimated in Neels (1981, p. 226).

[a]Not applicable.

Table A.2

COMPONENTS OF GROSS RENT

Input to Production of Housing Services	Green Bay, Wisconsin (1973)	South Bend, Indiana (1974)	Average
Annual Amount $ per Dwelling			
Land Input			
Current return[a]	126	78	102
Real estate tax[b]	41	24	33
Rent loss[c]	11	36	23
Total	178	138	158
Improvements Input			
Current return[a]	510	225	368
Real estate tax[b]	233	122	177
Rent loss[c]	50	105	78
Maintenance[d]	235	368	301
Insurance[e]	35	64	50
Total	1,063	884	974
Current Inputs			
Energy[f]	319	427	373
Other[g]	200	318	259
Total	519	745	632
Total gross rent	1,760	1,767	1,764
Share of Gross Rent			
Land input	.10	.08	.09
Improvements input	.61	.50	.55
Energy input	.18	.24	.21
Other current inputs	.11	.18	.15
Total gross rent	1.00	1.00	1.00

SOURCE: Housing Assistance Supply Experiment, baseline surveys of landlords and tenants.

[a]Estimated as a residual, gross rent less all other costs, allocated between land and improvements in proportion to their market values.

[b]Allocated between land and improvements in proportion to their assessed values.

[c]Losses due to vacancies and bad debts, allocated between land and improvements in proportion to their market values.

[d]Includes repairs and replacements, excludes capital additions, applies only to improvements.

[e]Only improvements are insured.

[f]Landlord and tenant payments for electricity, gas, fuel oil, and coal.

[g]Water and sewer charges, salaries of hired management and janitorial employees, and the imputed value of landlord labor.

Appendix B

ESTIMATION OF INVENTORY AND OCCUPANCY ELASTICITIES

This appendix estimates inventory and occupancy elasticities for rental housing using aggregated data from the 59 Standard Metropolitan Statistical Areas covered by the Annual Housing Survey (AHS). Those areas provide the data necessary to estimate the two elasticities because their housing markets vary widely in size and in short-run condition.

THEORY

A given housing market has associated with its current inventory of housing services a long-run equilibrium price of those services--the price at which producers of the services earn returns competitive with other investments and have no incentives to either expand or shrink the inventory. We assume that the relationship between long-run equilibrium price and inventory is

$$P^{*} = KH^{1/Y} , \qquad (B.1)$$

where P^{*} = long-run equilibrium price of housing services,

H = inventory of housing services,

K = constant (depends on units of measure),

Y = inventory elasticity (percentage change in inventory per one percent increase in long-run equilibrium price).

A demand for housing services is associated with the current market price of housing services. Since a one percent increase in price causes demand to decrease by 0.5 percent (see Sec. I), the relationship between demand and price is

$$D = AP^{-0.5} , \qquad (B.2)$$

where D = demand for housing services,

P = price of housing services

A = index of demand level (depends on units of measure).

If the housing market is in long-run equilibrium, then the amount of housing services demanded at the long-run equilibrium price will equal current inventory times long-run equilibrium occupancy rate (or "normal" occupancy rate). In other words, the price of housing services is set by the market to equate the demand for housing services with the product of inventory of housing services times occupancy rate (F):

$$D = FH . \qquad (B.3)$$

In markets where the level of demand (A) has remained stable long enough for inventory to adjust to it, the market clearing price (P) is equal to the long-run equilibrium price (P^*); and the occupancy rate (F) is equal to the long-run equilibrium occupancy rate (F^*). Let A^* denote the level of demand that would be in long-run equilibrium with the current inventory. Then, usings Eqs. (B.2) and (B.3), we find

$$F^* H = A^* P^{*-0.5} , \qquad (B.4)$$

where F^* = long-run equilibrium occupancy rate,

A^* = level of demand that would be in long-run equilibrium with current inventory.

Substituting Eq. (B.3) into Eq. (B.2) and dividing the result by Eq. (B.4) yields

$$\frac{F}{F^*} = (1 + x)\left(\frac{P}{P^*}\right)^{-0.5} , \qquad (B.5)$$

where $x = (A - A^*)/A^*$ = excess demand (fractional difference between current level that would be in long-run equilibrium with current inventory).

Equation (B.5) shows that if excess demand exists (i.e., if $x > 0$), then to clear the market either the occupancy rate (F) must be greater than the long-run equilibrium occupancy rate (F^*), or the current price (P) must be greater than the long-run equilibrium price (P^*), or both.

Estimating the proportion by which occupancy rate and price change as excess demand increases is one of the two objectives here, since their ratio is the occupancy elasticity. (The other objective is to estimate Y, the inventory elasticity.)

All we know theoretically about how occupancy rate and price behave as excess demand increases is that by definition the occupancy rate cannot exceed 1.0 (we also know that when excess demand is zero,

$P^* = P$ and $F^* = F$). To go beyond, we must appeal to evidence of housing-market behavior, limiting the unknowns so that the estimation task does not require more data than we have. The following formulation of the short-run relationship between occupancy rate and price as excess demand varies meets the requirements: it has only one unknown parameter (β); it obeys the constraint that occupancy rate always be less than 1.0; and it yields $P = P^*$ when $F = F^*$:

$$\frac{P}{P^*} = \left(\frac{F^{-10} - 1}{F^{*-10} - 1}\right)^{-\beta} \left(\frac{F}{F^*}\right)^{-2} , \qquad \text{(B.6)}$$

where β = parameters to be empirically determined.

Solving Eqs. (B.5) and (B.6) shows how occupancy rate and price vary with excess demand:[1]

$$F = F^* \left[F^{*10} + (1 - F^{*10})(1 + x)^{-2/\beta}\right]^{-0.1} ; \qquad \text{(B.7)}$$

and

$$P = P^* \left[(1 - F^{*10})(1 + x)^{10-2/\beta} + F^{*10}(1 + x)^{10}\right]^{0.2} . \qquad \text{(B.8)}$$

Applying the definitions of the price elasticity of occupancy rate to Eq. (B.6) then yields the final theoretical result:

---

[1] The specification in Eq. (B.6) was chosen partly because it makes possible an analytic solution to Eqs. (B.5) and (B.6).

$$Z = \left[\frac{10\beta}{1 - F^{10}} - 2\right]^{-1} , \qquad \text{(B.9)}$$

where Z = occupancy elasticity (percentage change in occupancy rate per one percent change in the ratio of current price to long-run equilibrium price).

ANALYSIS

To estimate the parameters Y (the inventory elasticity) and β (the value that gives the occupancy elasticity when substituted in Eq. (B.1)), we perform a regression analysis. Substituting Eq. (B.1) into Eq. (B.6) yields the relation between current price (P), current inventory (H), and current occupancy rate (F):

$$P = KH^{1/Y}\left(\frac{F^{-10} - 1}{F^{*-10} -1}\right)^{-\beta} \left(\frac{F}{F^*}\right)^{-2} . \qquad \text{(B.10)}$$

Dividing Eq. (B.10) by $(F/F^*)^{-2}$, then taking the logarithm, yields a regression equation that permits us to estimate 1/Y and β by linear regression analysis:

$$\ln\left(\frac{P}{(F/F^*)^{-2}}\right) = \ln\ (K) + \frac{1}{Y}\ln\ (H) - \beta\ \ln\left(\frac{F^{-10} - 1}{F^{*-10} - 1}\right) . \qquad \text{(B.11)}$$

The key to the regression analysis defined by Eq. (B.11) is to obtain an operational measure of the price of housing services (R).

Hedonic rent indexes, recently constructed for all 59 AHS areas by the Urban Institute, provide the required measure. In that formulation, the price of housing services in a particular metropolitan area equals the hedonic index for the area, evaluated for a reference bundle of housing and neighborhood characteristics (the reference is the average across all areas).[2]

We use the number of housing units in the metropolitan areas as the measure of market size (H), and 1.0 less the fraction of rental housing units that are vacant as the measure of rental occupancy rate (F). Both measures are readily available in published AHS reports, although neither is theoretically perfect.[3] Table B.1 lists the areas in question. Tables B.2 through B.4 report the data used in calculating our measures.

The long-run rental occupancy rate (F^*), needed to construct the variables in Eq. (B.9), is estimated as 1.0 minus the 20 year average of U.S. rental vacancy rates. Table B.5 shows that the average vacancy rate is 0.066, making the long-run equilibrium occupancy rate 0.934.

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[2] See Malpezzi, Ozanne, and Thibodeau (1980); and Follain and Malpezzi (1980) for a descriptions of the Urban Institute's extensive work on rent indexes. The institute never published the rents of the reference dwellings across the 59 metropolitan areas, but Larry Ozanne provided Rand with a copy of the computer printout generated by the institute's analysis. The rents are reported in the right-hand colums of Tables B.2 through B.4.

[3] The theoretical problem is that both H and F should be constructed using units of housing service rather than housing units. However, we can be sure that the theoretically correct measures are highly correlated with our operational measures; and with 59 areas, the sample is large enough to tolerate slight measurement errors.

Table B.1

STANDARD METROPOLITAN STATISTICAL AREAS INCLUDED IN THE ANNUAL HOUSING SURVEY OF SELECTED METROPOLITAN AREAS: BY YEAR OF SURVEY

1974	1975	1976
Albany-Schenectady-Troy, N.Y.	Atlanta, Ga.*	Allentown-Bethlehem-Easton, Pa.-N.J.
Anaheim-Santa Ana-Garden Grove, Calif.	Chicago, Ill.*	Baltimore, Md.
Boston, Mass.*	Cincinnati, Ohio-Ky.-Ind.	Birmingham, Ala.
Dallas, Tex.	Colorado Springs, Colo.	Buffalo, N.Y.
Detroit, Mich.*	Columbus, Ohio	Cleveland, Ohio
Fort Worth, Tex.	Hartford, Conn.	Denver, Colo.
Los Angeles-Long Beach, Calif.*	Kansas City, Mo.-Kans.	Grand Rapids, Mich.
Memphis, Tenn.-Ark.	Madison, Wis.	Honolulu, Hawaii
Minneapolis-St. Paul, Minn.	Miami, Fla.	Houston, Tex.*
Newark, N.J.	Milwaukee, Wis.	Indianapolis, Ind.
Orlando, Fla.	New Orleans, La.	Las Vegas, Nev.
Phoenix, Ariz.	Newport News-Hampton, Va.	Louisville, Ky.-Ind.
Pittsburgh, Pa.	Paterson-Clifton-Passaic, N.J.	New York, N.Y.*
Salt Lake City, Utah	Philadelphia, Pa.-N.J.*	Oklahoma City, Okla.
Spokane, Wash.	Portland, Oreg.-Wash.	Omaha, Nebr.-Iowa
Tacoma, Wash.	Rochester, N.Y.	Providence-Pawtucket-Warwick, R.I.-Mass.
Washington, D.C.-Md.-Va.*	San Antonio, Tex.	Raleigh, N.C.
Wichita, Kans.	San Bernardino-Riverside-Ontario, Calif.	Sacramento, Calif.
	San Diego, Calif.	St. Louis, Mo.-Ill.*
	San Francisco-Oakland, Calif.*	Seattle-Everett, Wash.*
	Springfield-Chicopee-Holyoke, Mass.-Conn.	

NOTE: Surveys of individual housing units were spread evenly over a year beginning in April and extending through March of the following calendar year. On average the survey reflects condition in October of the survey year.

*Sample size is 15,000 housing units; all other samples are 5,000 housing units.

Table B.2

HOUSING-MARKET CHARACTERISTICS FOR SELECTED METROPOLITAN AREAS: OCTOBER 1974

Standard Metropolitan Statistical Area (SMSA)	Total Housing Units[a] (000)	Rental Vacancy Rate[b] (%)	Renter Households Moved In Within Past 12 Months[c] (%)	Price of Rental Housing Services[d] ($/Unit/Mo.)
Albany	261	8.3	30.8	125.25
Anaheim	592	8.1	51.1	150.13
Boston	931	7.0	28.9	156.46
Dallas	640	13.9	54.6	115.02
Detroit	1,423	10.7	33.3	129.56
Fort Worth	293	10.9	52.7	94.75
Los Angeles	2,699	7.4	38.1	155.65
Memphis	286	11.0	21.0	99.72
Minneapolis	641	6.0	49.4	140.86
Newark	608	3.8	21.0	174.49
Orlando	200	23.1	51.8	125.73
Phoenix	463	14.4	57.2	118.87
Pittsburgh	823	5.1	26.6	114.89
Salt Lake City	197	8.1	51.0	111.86
Spokane	112	9.0	51.9	108.70
Tacoma	148	5.9	52.6	112.09
Washington, D.C.	1,039	4.9	31.6	149.95
Wichita	139	7.2	46.9	101.29

SOURCE: U.S. Bureau of the Census (1974).

[a]All year-round housing units in SMSA; includes both owner-occupied and renter-occupied housing units.

[b]Percent of rental units vacant and for rent; excludes units rented but not occupied.

[c]Percent of renter-occupied units whose occupants moved in during the past 12 months.

[d]Rent of a reference housing unit; Urban Institute hedonic index of rent, evaluated at a reference bundle of dwelling and neighborhood characteristics.

Table B.3

HOUSING-MARKET CHARACTERISTICS FOR SELECTED METROPOLITAN AREAS: OCTOBER 1975

Standard Metropolitan Statistical Area (SMSA)	Total Housing Units[a] (000)	Rental Vacancy Rate[b] (%)	Renter Households Moved In Within Past 12 Months[c] (%)	Price of Rental Housing Services[d] ($/Unit/Mo.)
Atlanta	569	15.0	44.1	124.00
Chicago	2,422	6.7	27.7	156.15
Cincinnati	470	7.0	37.6	113.37
Colorado Springs	109	17.6	60.2	120.78
Columbus	334	10.8	40.2	108.89
Hartford	230	6.8	25.8	148.59
Kansas City	461	11.3	45.1	112.75
Madison	109	5.2	50.4	130.40
Miami	560	11.1	40.8	159.83
Milwaukee	481	3.6	33.5	138.80
New Orleans	383	7.8	35.7	111.05
Newport News	101	13.2	48.6	109.44
Paterson	447	3.3	21.3	190.37
Philadelphia	1,621	7.0	30.4	142.14
Portland	417	7.1	48.7	135.31
Rochester	302	8.6	37.5	157.10
San Antonio	293	9.6	47.6	112.05
San Bernardino	461	10.3	46.5	117.65
San Diego	579	5.7	49.7	151.79
San Francisco	1,249	7.2	37.8	175.84
Springfield	179	6.5	33.6	132.35

SOURCE: U.S. Bureau of the Census (1975a).

[a]All year-round housing units in SMSA; includes both owner-occupied and renter-occupied housing units.

[b]Percent of rental units vacant and for rent; excludes units rented but not occupied.

[c]Percent of renter-occupied units whose occupants moved in during the past 12 months.

[d]Rent of a reference housing unit; Urban Institute hedonic index of rent, evaluated at a reference bundle of dwelling and neighborhood characteristics.

Table B.4

HOUSING-MARKET CHARACTERISTICS FOR SELECTED METROPOLITAN AREAS: OCTOBER 1976

Standard Metropolitan Statistical Area (SMSA)	Total Housing Units[a] (000)	Rental Vacancy Rate[b] (%)	Renter Households Moved In Within Past 12 Months[c] (%)	Price of Rental Housing Services[d] ($/Unit/Mo.)
Allentown	203	4.8	29.3	141.16
Baltimore	729	5.2	29.6	135.39
Birmingham	277	6.2	35.3	101.28
Buffalo	454	4.4	28.4	140.61
Cleveland	718	5.5	29.3	132.68
Denver	545	8.8	52.1	138.14
Grand Rapids	190	6.1	44.4	129.42
Honolulu	219	6.0	39.8	206.31
Houston	834	8.0	48.2	146.79
Indianapolis	410	8.2	40.8	120.82
Las Vegas	132	7.4	52.8	154.34
Louisville	305	10.2	38.1	113.81
New York	4,041	5.3	17.3	187.06
Oklahoma City	273	12.8	54.5	110.70
Omaha	200	8.8	41.5	116.77
Providence	327	6.6	27.1	144.95
Raleigh	95	4.1	42.3	114.87
Sacramento	348	6.5	51.7	134.99
St. Louis	812	7.3	33.3	123.65
Seattle	554	5.9	45.6	149.44

SOURCE: U.S. Bureau of the Census (1976).

[a]All year-round housing units in SMSA; includes both owner-occupied and renter-occupied housing units.

[b]Percent of rental units vacant and for rent; excludes units rented but not occupied.

[c]Percent of renter-occupied units whose occupants moved in during the past 12 months.

[d]Rent of a reference housing unit; Urban Institute hedonic index of rent, evaluated at a reference bundle of dwelling and neighborhood characteristics.

Table B.5

RENTAL VACANCY RATES FOR THE UNITED STATES:
1960 TO 1979

Year	First Quarter	Second Quarter	Third Quarter	Fourth Quarter	Annual Average
1979	4.8	5.0	5.2	5.0	5.0
1978	5.0	5.1	5.0	5.0	5.0
1977	5.1	5.3	5.4	5.1	5.2
1976	5.5	5.8	5.7	5.3	5.6
1975	6.1	6.3	6.2	5.4	6.0
1974	6.2	6.3	6.2	6.0	6.2
1973	5.7	5.8	5.8	5.8	5.8
1972	5.3	5.5	5.8	5.6	5.6
1971	5.3	5.3	5.6	5.6	5.5
1970	5.4	5.4	5.3	5.2	5.3
1969	5.6	5.7	5.5	5.1	5.5
1968	6.1	6.2	5.9	5.4	5.9
1967	7.3	6.9	7.0	6.2	6.9
1966	8.3	7.4	7.4	7.7	8.3
1965	8.5	8.2	7.8	8.5	8.3
1964	8.1	8.1	8.4	8.3	8.2
1963	8.0	8.2	8.3	8.3	8.2
1962	8.5	8.1	8.0	8.1	8.2
1961	8.9	8.8	8.6	8.5	8.7
1960	8.0	8.0	8.3	8.4	8.2
20 year average	6.6	6.6	6.6	6.4	6.6

SOURCE: U.S. Bureau of the Census (1979, p. 1).

Constructing the dependent[4] and independent variables in Eq. (B.11) from the data in Tables B.2, B.3, and B.4 (using $F^* = 0.934$) and performing a linear regression analysis yields the following parameter estimates:

Parameter	Point Estimate	Standard Error	t-Ratio
ln(k)	4.3097	--	--
1/Y	.0886	.0223	3.98
β	.2006	.0285	7.04

NOTE: $R^2 = 0.582$; R^2 (adjusted) = 0.567.

The high R^2 (fraction variation explained) and the high t-ratios (ratio of point estimates to their standard errors) indicate that the regression equation fits the data very well. However, only an analysis of residuals can show whether the model is adequately specified.

Tables B.6 and B.7 report a detailed analysis of residuals. The predicted long-run equilibrium price in Table B.6 comes from Eq. (B.7), with the occupancy rate (F) set equal to its long-run equilibrium level of 0.066. The actual long-run equilibrium price in Table B.6 was obtained by adjusting observed price for deviations of the observed occupancy rate from its long-run equilibrium level. Similarly, the predicted short-run equilibrium price in Table B.7 comes from Eq. (B.7), with market size set equal to a reference level of 500 housing

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[4] For the regression analysis, the housing price in Tables B.3 and B.4 were adjusted to 1974 dollars using the national rent index in the President's 1980 economic report (U.S. Government Printing Office, 1980, p. 262). Specifically, the 1975 nominal prices were divided by 1.051, and the 1976 nominal prices by 1.108.

Table B.6

PREDICTED VS. ACTUAL LONG-RUN EQUILIBRIUM RENTS

SMSA		Total Housing Units	Long Run Equilibrium Price of Housing Services ($/Unit/Mo)		
Rank	Name	(000)	Predicted	Actual	Difference
1	Raleigh	95.0	111.6	96.3	15.3
2	Newport News	101.0	112.2	113.4	-1.2
3	Madison	109.0	113.0	119.7	-6.7
4	Colorado Springs	109.0	113.0	128.2	-15.2
5	Spokane	112.0	113.3	113.4	-.1
6	Las Vegas	132.0	114.9	141.6	-26.6
7	Wichita	139.0	115.5	102.6	12.9
8	Tacoma	148.0	116.1	110.3	5.8
		118.1	113.7	115.7	-2.0
9	Springfield	179.0	118.1	125.7	-7.6
10	Grand Rapids	190.0	118.7	115.5	3.2
11	Salt Lake City	197.0	119.1	115.1	4.0
12	Omaha	200.0	119.3	109.6	9.7
13	Orlando	200.0	119.3	142.6	-23.3
14	Allentown	203.0	119.4	121.4	-2.0
		194.8	119.0	121.6	-2.7
15	Honolulu	219.0	120.2	183.6	-63.4
16	Hartford	230.0	120.7	142.0	-21.2
17	Albany	261.0	122.1	129.3	-7.2
18	Oklahoma City	273.0	122.6	108.5	14.1
19	Birmingham	277.0	122.8	90.6	32.2
20	Memphis	286.0	123.1	106.5	16.6
		257.7	121.9	126.7	-4.8
21	San Antonio	293.0	123.4	112.1	11.3
22	Fort Worth	293.0	123.4	101.1	22.3
23	Rochester	302.0	123.7	155.0	-31.3
24	Louisville	305.0	123.8	108.8	15.0
25	Providence	327.0	124.6	130.8	-6.2
26	Columbus	334.0	124.8	110.4	14.4
		309.0	124.0	119.7	4.2
27	Sacramento	348.0	125.3	121.6	3.7
28	New Orleans	383.0	126.4	108.2	18.2
29	Indianapolis	410.0	127.1	112.4	14.8
30	Portland	417.0	127.3	130.1	-2.8
31	Paterson	447.0	128.1	162.2	-34.1
32	Buffalo	454.0	128.3	119.2	9.0
		409.8	127.1	125.6	1.5
33	San Bernardino	461.0	128.5	118.7	9.8
34	Kansas City	461.0	128.5	114.9	13.5
35	Phoenix	463.0	128.5	130.5	-2.0
36	Cincinnati	470.0	128.7	108.8	19.9
37	Milwaukee	481.0	128.9	120.0	8.9
38	Denver	545.0	130.4	129.7	.7
		480.2	128.9	120.4	8.5
39	Seattle	554.0	130.6	132.7	-2.1
40	Miami	560.0	130.7	162.6	-31.9
41	Atlanta	569.0	130.9	130.0	.9
42	San Diego	579.0	131.1	141.3	-10.2
43	Anaheim	592.0	131.3	154.5	-23.1
44	Newark	608.0	131.7	160.0	-28.4
		577.0	131.0	146.9	-15.8
45	Dallas	640.0	132.3	125.9	6.4
46	Minneapolis	641.0	132.3	138.9	-6.6
47	Cleveland	718.0	133.6	116.5	17.1
48	Baltimore	729.0	133.8	117.9	15.9
49	St. Louis	812.0	135.1	113.2	21.9
		708.0	133.4	122.5	10.9
50	Pittsburgh	823.0	135.3	110.5	24.7
51	Houston	834.0	135.4	136.1	-.7
52	Boston	931.0	136.7	157.8	-21.0
53	Washington, D.C.	1039.0	138.1	143.3	-5.3
54	San Francisco	1249.0	140.4	169.4	-29.0
		975.2	137.2	143.4	-6.2
55	Detroit	1423.0	142.0	138.0	4.0
56	Philadelphia	1621.0	143.7	136.4	7.3
57	Chicago	2422.0	148.9	148.9	.0
58	Los Angeles	2699.0	150.3	158.2	-7.9
59	New York	2441.2	148.1	149.0	.8

Table B.7

PREDICTED VS. ACTUAL SHORT-RUN EQUILIBRIUM PRICES

SMSA		Occupancy Rate	Short Run Equilibrium Price of Housing Services ($/Year/Mo)		
Rank	Name	(%)	Predicted	Actual	Difference
1	Orlando	76.9	104.8	125.7	-21.0
2	Colorado Springs	82.4	106.5	121.3	-14.8
3	Atlanta	85.0	107.8	107.5	.3
4	Phoenix	85.6	108.2	110.3	-2.1
5	Dallas	86.1	108.6	103.7	4.8
6	Newport News	86.8	109.1	110.7	-1.6
7	Oklahoma City	87.2	109.4	97.2	12.2
8	Kansas City	88.7	110.9	99.6	11.3
		84.8	108.1	109.5	-1.4
9	Miami	88.9	111.1	138.8	-27.7
10	Memphis	89.0	111.2	96.6	14.6
11	Fort Worth	89.1	111.3	91.6	19.7
12	Columbus	89.2	111.4	99.0	12.4
13	Detroit	89.3	111.5	108.8	2.7
14	San Bernardino	89.7	112.0	103.9	8.1
		89.2	111.4	106.4	5.0
15	Louisville	89.8	112.2	98.9	13.2
16	San Antonio	90.4	113.0	103.1	9.9
17	Spokane	91.0	113.9	114.5	-.6
18	Omaha	91.2	114.2	105.4	8.8
19	Denver	91.2	114.2	114.0	.2
20	Rochester	91.4	114.6	144.1	-29.5
		90.8	113.7	113.3	.3
21	Albany	91.7	115.1	122.3	-7.2
22	Indianapolis	91.8	115.3	102.3	13.0
23	Salt Lake City	91.9	115.5	112.0	3.4
24	Anaheim	91.9	115.5	136.3	-20.9
25	Houston	92.0	115.6	116.7	-1.0
26	New Orleans	92.2	116.0	99.7	16.3
		91.9	115.5	114.9	.6
27	Las Vegas	92.6	116.9	144.5	-27.7
28	Los Angeles	92.6	116.9	123.5	-6.6
29	St. Louis	92.7	117.1	98.5	18.6
30	San Francisco	92.8	117.3	142.1	-24.8
31	Wichita	92.8	117.3	104.6	12.7
32	Portland	92.9	117.6	120.6	-3.0
		92.7	117.2	122.3	-5.1
33	Philadelphia	93.0	117.8	112.3	5.5
34	Cincinnati	93.0	117.8	100.0	17.8
35	Boston	93.0	117.8	136.4	-18.7
36	Hartford	93.2	118.3	139.6	-21.4
37	Chicago	93.3	118.5	119.0	-.5
38	Providence	93.4	118.8	125.2	-6.4
		93.2	118.2	122.1	-3.9
39	Sacramento	93.5	119.0	116.0	3.1
40	Springfield	93.5	119.0	127.2	-8.1
41	Birmingham	93.8	119.9	88.8	31.1
42	Grand Rapids	93.9	120.2	117.3	2.8
43	Honolulu	94.0	120.5	184.7	-64.2
44	Minneapolis	94.0	120.5	127.0	-6.5
		93.8	119.8	126.8	-7.0
45	Seattle	94.1	120.8	123.2	-2.4
46	Tacoma	94.1	120.8	115.1	5.6
47	San Diego	94.3	121.4	131.4	-10.0
48	Cleveland	94.5	122.0	106.9	15.2
49	New York	94.7	122.7	129.2	-6.5
		94.3	121.5	121.2	.4
50	Baltimore	94.8	123.1	108.9	14.2
51	Madison	94.8	123.1	131.0	-7.8
52	Pittsburgh	94.9	123.5	101.3	22.2
53	Washington, D.C.	95.1	124.3	129.5	-5.2
54	Allentown	95.2	124.7	127.2	-2.6
		95.0	123.7	119.6	4.1
55	Buffalo	95.6	126.4	118.0	8.4
56	Raleigh	95.9	127.9	110.8	17.1
57	Newark	96.2	129.5	158.1	-28.5
58	Milwaukee	96.4	140.7	122.1	8.5
59	Paterson	96.7	132.6	168.6	-36.0
		96.2	129.4	135.5	-6.1

units. The actual short-run equilibrium price was obtained by adjusting the observed price for deviations of the observed market size from the reference level.

Figures B.1 and B.2 graph the findings shown in Tables B.6 and B.7. They plot the predicted prices as a curve and aggregate the actual prices into ten points. Both figures indicate that our model of housing price is adequately specified in that there is no systematic deviation between actual and predicted prices.

As is usual in regression analysis, however, the residuals analysis reveals considerable imperfection in the model. Even the aggregated actual prices deviate from the predictions, and the actual prices in specific metropolitan areas deviate considerably from the predictions (see Tables B.6 and B.7). We attempted to improve the model by adding renter mobility rate as a third independent variable (specifically, we added the logarithm of the percent of rental households who had moved during the previous 12 months). However, no improvement resulted--R^2 remained the same and adjusted R^2 fell to 0.561. The coefficient of the added variable was -0.023, with a standard error of 0.080 and a t-ratio of only -0.29.

CONCLUSIONS

Inverting the estimate $1/Y = .0886$, we conclude that the inventory elasticity (Y) equals 11.3. Substituting the estimate $\beta = 0.20$ into Eq. (B.9), we find that occupancy elasticity varies with current occupancy rate:

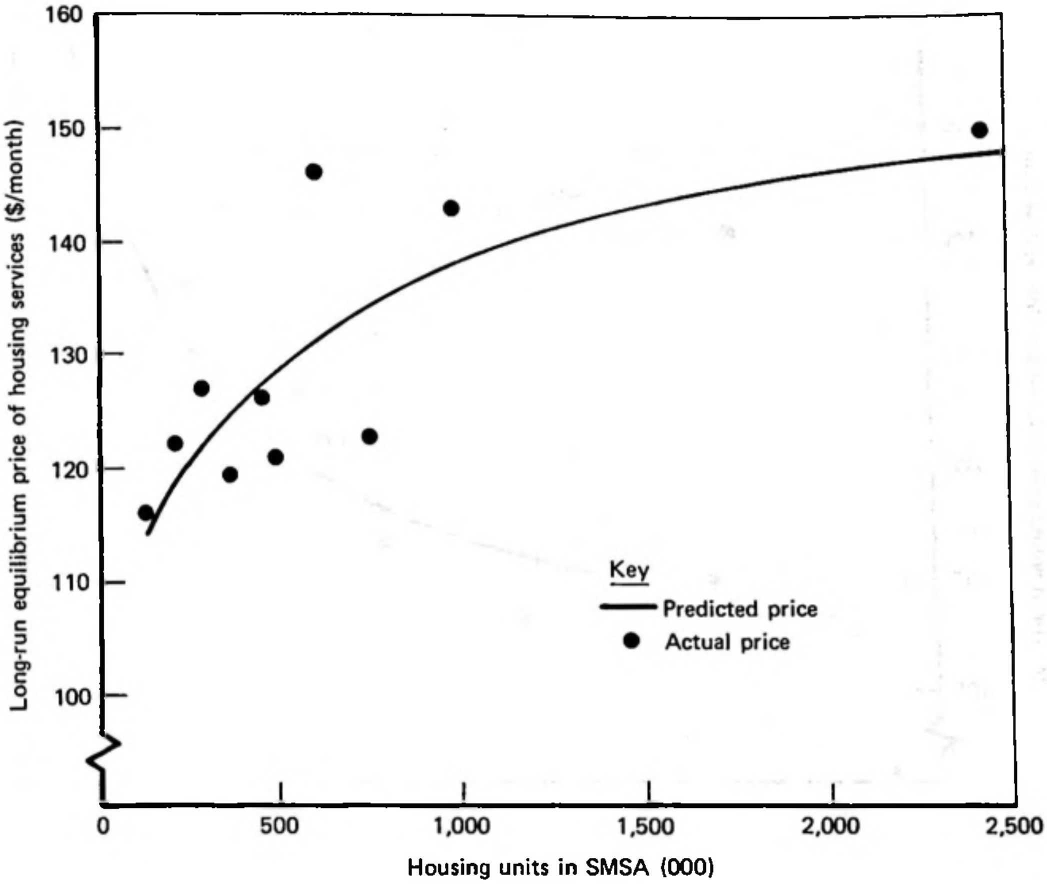

Fig. B.1--Supply curve

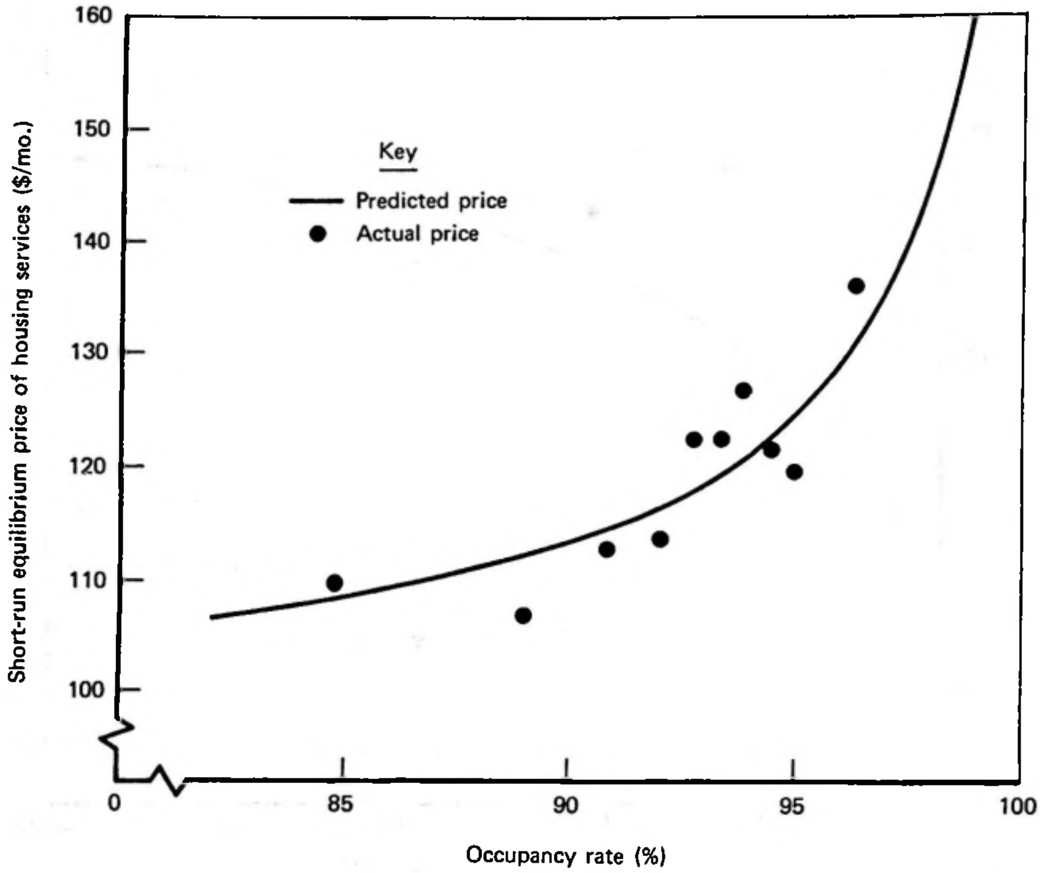

Fig. B.2--Occupied supply curve

$$Z = 0.5\ [F^{-10} - 1]\ , \qquad \text{(B.12)}$$

where Z = occupancy elasticity,

F = current occupancy rate.

Substituting β = 0.20 and F^* = 0.934 into Eqs. (B.7) and (B.8) shows how rental occupancy rate and price react to excess demand:

$$\frac{F}{F^*} = [0.505 + 0.495\ (1 + x)^{-10}]^{-0.1}\ , \qquad \text{(B.13)}$$

$$\frac{P}{P^*} = [0.495 + 0.505\ (1 + x)^{10}]^{0.2}\ , \qquad \text{(B.14)}$$

where F = rental occupancy rate,

P = price of housing service,

x = fraction excess demand.

Table 2 (Sec. II) tabulates Eq. (B.12) to show the occupancy elasticity corresponding to various occupancy rates. Table 3 (also in Sec. II) tabulates Eqs. (B.13) and (B.14) to show the effect of various amounts of excess demand on occupancy rate and price.

Appendix C

HOUSING-MARKET RESPONSE TO DEMAND INCREASE

This appendix derives the results presented in Sec. III by building a model of housing-market response to increases in demand. By changing the sign of the demand shift, the model also gives market responses to decreases in demand.

To make the model generally useful, we specify that demand (evaluated at initial market price) grows to a new level at a finite pace. To determine the consequences of an instantaneous demand shift, we make the pace of demand adjustment infinite:

$$d'(t) = \alpha[d - d(t)] \ , \qquad (C.1)$$

where $d(t)$ = relative increase in demand as a function of years (t) since increase began,

d = ultimate relative increase in demand,

α = pace of demand adjustment (fraction of gap between ultimate and current increase closed per year).

Solving Eq. (C.1) using the initial condition d(0) yields increased demand as a function of time:

$$d(t) = d\ [1 - e^{-\alpha t}] \ . \qquad (C.2)$$

LONG-RUN EQUILIBRIUM

Any given demand increase implies a repair response, an inventory response, and an increase in the long-run equilibrium price that would eventually occur if there were no additional demand changes. The long-run market responses to a demand increase comprise three relations. First, the change in supply equals the change in demand less the price change times the price elasticity of demand:

$$k^*(t) + i^*(t) = d(t) - S\ c(t)\ , \tag{C.3}$$

where $k^*(t)$ = long-run relative increase in housing services due to repair of existing housing,

$i^*(t)$ = long-run relative increase in housing services due to repairs,

$c(t)$ = long-run relative increase in price of housing services,

S = price elasticity of demand (percentage decrease in demand per one percent increase in price).

Second, the increase in housing services from existing housing equals the repair elasticity times the price increase:

$$k^*(t) = W\ c(t)\ , \tag{C.4}$$

where W = price elasticity of housing services due to repairs (percentage increase in supply of housing services over existing supply per one percent increase in price).

Third, the increase in housing inventory equals the inventory elasticity times the price change:

$$i^*(t) = Y\ c(t)\ , \tag{C.5}$$

where Y = price elasticity of housing services due to inventory change (percentage increase in inventory per one percent increase in price).

Solving Eqs. (C.3) - (C.5) yields the following relations:

$$c(t) = \left[\frac{1}{W + Y + S}\right] d(t) \ , \tag{C.6}$$

$$k^*(t) = \left[\frac{W}{W + Y + S}\right] d(t) \ , \tag{C.7}$$

and

$$i^*(t) = \left[\frac{Y}{W + Y + S}\right] d(t) \ . \tag{C.8}$$

Neither the repair responses (see Rydell and Neels, 1982) nor the inventory responses (see Muth, 1960) to increased demand occur instantly. Rather, they close the gap between desired supply and current supply at a finite pace of adjustment each year. For the repair response, the adjustment is expressed as

$$k'(t) = \gamma[k^*(t) - k(t)] \ , \tag{C.9}$$

where k(t) = relative increase in housing services due to repair of existing housing that occurs by year t,

γ = pace of repair response (fraction of gap between desired and current levels of housing service closed per year).

For the inventory response, the adjustment is expressed as

$$i'(t) = \beta[i^*(t) - i(t)] \ , \qquad (C.10)$$

where i(t) = relative increase in housing inventory that occurs by year t,

β = pace of inventory adjustment (fraction of gap between desired and actual inventory closed per year).

Substituting Eq. (C.2) into Eqs. (C.7) and (C.8) and the results into Eqs. (C.9) and (C.10), then solving the differential equations with initial conditions k(0) = i(0) = 0 yields

$$k(t) = \frac{Wd}{W + Y + S}\left[1 + \left(\frac{\gamma}{\alpha - \gamma}\right)e^{-\alpha t} - \left(\frac{\alpha}{\alpha - \gamma}\right)e^{-\gamma t}\right] \qquad (C.11)$$

and

$$i(t) = \frac{Yd}{W + Y + S}\left[1 + \left(\frac{\beta}{\alpha - \beta}\right)e^{-\alpha t} - \left(\frac{\alpha}{\alpha - \beta}\right)e^{-\beta t}\right] . \qquad (C.12)$$

SHORT-RUN EQUILIBRIUM

During the time it takes inventory to adjust to its long-run equilibrium level, temporary increases in occupancy rates and price clear the market. Such short-run changes comprise two relations. First, the change in supply of housing services equals the change in demand less the price change times the price elasticity of demand:

$$k(t) + i(t) + f(t) = d(t) - S\ p(t) \ , \qquad (C.13)$$

where f(t) = relative increase in occupancy of existing housing,

p(t) = short-run relative increase in price of housing services.

Second, the increase in occupancy equals the differential short-run price increase times the occupancy elasticity:

$$f(t) = Z[p(t) - c(t)] \ , \tag{C.14}$$

where Z = price elasticity of occupancy rate (percentage increase in occupancy rate per one percent increase in short-run price relative to long-run price).

(Note that the occupancy increase [f(t)] is proportional to the gap between the short-run price increase [p(t)] and the long-run price increase [c(t)]. When the housing market has fully adjusted to the demand increase, the two price increases will be the same. The occupancy-rate change will therefore be zero, and Eq. (C.13) will be identical to Eq. (C.3)). Solving Eqs. (C.13) and (C.14) and using Eq. (C.6) yields

$$f(t) = \left[\frac{Z}{Z + S}\right]\left\{\left(\frac{W + Y}{W + Y + S}\right) d(t) - k(t) - i(t)\right\} \tag{C.15}$$

and

$$p(t) = \left[\frac{1}{S}\right][d(t) - k(t) - i(t) - f(t)] \ . \tag{C.16}$$

COMPOSITE SUPPLY ELASTICITY

By definition, the price elasticity of supply equals the ratio of the relative increase in total supply to the relative increase in price--[k(t) + i(t) + f(t)]/p(t). Using Eqs. (C.2), (C.11), (C.15), and (C.16), the composite supply elasticity can be evaluated for any time (t) since the demand increase began.

However, such evaluation requires estimates of the parameters in the equations. Using the parameter values given in Table C.1 we predict that the market will respond to an instantanous increase in demand (d) first, by upgrading (repairing) existing housing:

$$k(t) = 0.0166[d][1 - e^{-0.08t}] \; ; \qquad (C.17)$$

second, by increasing inventory:

$$i(t) = 0.9417[d][1 - e^{-0.32t}] \; ; \qquad (C.18)$$

third, by increasing occupancy:

$$f(t) = [1 - F^{10}] \{0.9583[d] - k(t) - i\,(t)\} \; ; \qquad (C.19)$$

and fourth, by increasing price:

$$p(t) = 2[d - k(t) - i(t) - f(t)] \; . \qquad (C.20)$$

Equations (C.17) to (C.20) were used to fill in Table 4 (Sec. III). We modeled the short run using $t = 0$, and the long run using $t = \infty$.

Table C.1

PARAMETER ESTIMATES FOR HOUSING-MARKET MODEL

Symbol	Definition	Estimate
W	Repair elasticity[a]	.2
Y	Inventory elasticity[b]	11.3
Z	Occupancy elasticity[c]	$.5[F^{10}-1]$
S	Price elasticity of demand[d]	.5
α	Pace of demand increase[e]	∞
β	Pace of inventory response[f]	.32
γ	Pace of repair response[g]	.08

[a]From Rydell and Neels (1982).

[b]From Appendix B, this report.

[c]From Appendix B, this report; F = initial rental occupancy rate.

[d]Central tendency in Mayo's (1981) literature review.

[e]Assumes instant demand shift.

[f]From Muth (1960).

[g]From Rydell and Neels (1982).

Appendix D

HOUSING-MARKET CONDITION ESTIMATED FROM RENTAL VACANCY RATE

A problem with using vacancy rates to measure housing-market condition is that they are affected by household mobility as well as by excess demand. This appendix shows how the results given in Table 7 (Sec. IV) must be modified when the renter mobility rate in a particular housing market differs from the national average.

The vacancy rate is an increasing function of household mobility: the greater the mobility, the more vacancies are occurring at any time. Regressing observed vacancy rates on mobility rates (using data for the 59 metropolitan areas selected for the Annual Housing Survey—see Tables B.2 to B.4, Appendix B) yields

$$W^* = 0.1484\ m^{0.71} , \tag{D.1}$$

where W^* = long-run equilibrium rental vacancy rate,

m = fraction of renter households that moved in during previous 12 months.

The standard error of estimate on the power parameter (0.71) is 0.15. The log-linear regression explains only 26 percent of the variance in vacancy rates because mobility rate affects just the long-run equilibrium vacancy rate, not short-run deviations. In other words, the observed vacancy rate is a very "noisy" estimate of the long-run equilibrium rate. Fortunately, noise in the independent variable of a regression equation does not bias the parameter estimates.

Adjusting the intercept parameter in Eq. (D.1) slightly to give the national average long-run equilibrium vacancy rate when the renter mobility rate equals its national average,[1] and transforming the equation to predict the long-run equilibrium occupancy rate as a function of the mobility rate, yields

$$F^* = 1 - 0.066 \left[\frac{m}{0.35}\right]^{0.71} , \qquad (D.2)$$

where F^* = long-run equilibrium rental occupancy rate $(1 - W^*)$. Then, using Eqs. (B.7) and (B.8) and the estimate β = 0.20 from Appendix B, we obtain

$$F = F^*[F^{*10} + (1 - F^{*10})(1 + x)^{-10}]^{-0.1} \qquad (D.3)$$

and

$$P = P^*[1 - F^{*10} + F^{*10} (1 + x)^{10}]^{0.2} , \qquad (D.4)$$

where F = rental occupancy rate (1.0 minus vacancy rate),

P/P^* = price of rental housing services relative to long-run equilibrium price,

x = fraction excess demand (zero in long-run equilibrium).

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[1] The national average long-run equilibrium vacancy rate, 0.066, is estimated in Table B.1 in Appendix B. The national average renter mobility rate, 0.35 (fraction moved in during previous 12 months), comes from U.S. Bureau of the Census (1975b).

Table D.1 was constructed by substituting Eq. (D.2) into Eqs. (D.3) and (D.4). The central column shows how vacancy rate and price vary with excess demand when the mobility rate equals the national average (0.35). (The same results were presented earlier in Table 7, Sec. IV.) The other columns report the results for other mobility rates.

The table also shows how the mobility rate in a housing market affects the way the vacancy rate indicates market condition. For example, if the mobility rate is 0.35, then a 5.0 percent vacancy rate indicates 4 percent of excess demand (see top half of table, fifth line) and 4.4 percent of extra price (see bottom half of table, fifth line). However, if the mobility rate is 0.25, then a 5.0 percent vacancy rate indicates essentially no excess demand and no extra price (see sixth lines in the top and bottom halves of the table).

Table D.1

EFFECT OF EXCESS DEMAND AND HOUSEHOLD MOBILITY ON VACANCY RATE AND PRICE

Excess Demand (%)[a]	Fraction Renter Households Moved In During Previous 12 Months				
	.25	.30	.35	.40	.45
	Rental Vacancy Rate (%)[b]				
20	1.1	1.3	1.5	1.7	1.9
16	1.5	1.7	2.0	2.0	2.5
12	2.0	2.4	2.7	3.0	3.4
8	2.8	3.2	3.7	4.1	4.5
4	3.8	4.4	5.0	5.5	6.0
0	5.2	5.9	6.6	7.3	7.9
-4	7.0	7.8	8.7	9.4	10.2
-8	9.2	10.2	11.1	12.0	12.8
-12	11.9	13.0	14.0	14.9	15.8
-16	14.9	16.1	17.2	18.2	19.1
-20	18.3	19.6	20.7	21.6	22.5
	Extra Price (%)[c]				
20	32.2	30.8	29.4	28.1	26.8
16	24.6	23.3	22.2	21.1	20.1
12	17.4	16.5	15.6	14.8	14.0
8	10.9	10.3	9.7	9.1	8.6
4	5.1	4.7	4.4	4.2	3.9
0	.0	.0	.0	.0	.0
-4	-4.3	-3.9	-3.6	-3.4	-3.1
-8	-7.7	-7.1	-6.5	-6.0	-5.6
-12	-10.4	-9.5	-8.7	-8.0	-7.3
-16	-12.4	-11.2	-10.2	-9.4	-8.6
-20	-13.8	-12.4	-11.3	-10.3	-9.5

SOURCE: Equations (D.2) - (D.4).

[a]Percentage difference between the current level of demand for housing services and the level that would be in long-run equilibrium with the current inventory of housing services.

[b]Vacant dwellings for rent, as a percent of occupied rental dwellings plus vacant dwellings for rent (U.S. Census definition of "rental vacancy rate."

[c]Percentage deviation of the current price of housing services from the long-run equilibrium price.

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