RAND/WN-10074-HUD

VACANCY DURATION AND HOUSING MARKET CONDITION

C. Peter Rydell

January 1978

This Note was prepared for the DEPARTMENT OF HOUSING AND URBAN DEVELOPMENT, under Contract No. H-1789. It is intended to facilitate communication of preliminary research results. Views or conclusions expressed herein may be tentative and do not represent the official opinion of the sponsoring agency.

Rand
SANTA MONICA, CA. 90406

PREFACE

This working note was prepared for the Office of Policy Development and Research, U.S. Department of Housing and Urban Development (HUD). It analyzes a measure of housing market condition that, in cross-sectional analyses, is superior to the traditional measure, vacancy rate.

The note uses data from the Current Population Survey, the Quarterly Household Survey, and the Annual Housing Survey, all conducted by the U.S. Bureau of the Census, to draw conclusions about national housing market conditions. It uses data from the baseline surveys of the Housing Assistance Supply Experiment, conducted by The Rand Corporation under contract to HUD, to contrast the market conditions in the experiment's two sites: Brown County, Wisconsin, and St. Joseph County, Indiana.

Kevin McCarthy helped estimate turnover rates. Ira S. Lowry and John Mulford reviewed the entire draft, improving both substance and presentation. Christine D'Arc edited the text. Judy Arreola and Barbara Wilson typed the draft, and Robin Boynton and Joan Pederson typed the final copy.

SUMMARY

The traditional measure of housing market condition is the vacancy rate. This working note proposes another measure: the annual average duration of a housing vacancy. The two measures are related by the turnover rate. If the turnover rate is stable, as in most longitudinal analyses of a given housing market, vacancy rate and vacancy duration are equally satisfactory measures of housing market condition. If the turnover rate varies, however, as it does in cross-sectional analyses of different markets and submarkets, the vacancy rate and vacancy duration differ, and empirical evidence shows that vacancy duration is the better measure.

To evaluate the performance of vacancy rate and vacancy duration, we use the market value of a unit of housing capital as an operational definition of market condition: The higher the market value, the tighter the market. Data from the Housing Assistance Supply Experiment (HASE) show that average vacancy duration is a better predictor of market value than vacancy rate.

The note shows that the annual average vacancy rate is the product of annual turnover rate and annual average vacancy duration. Apparently, turnover rate is a demographic characteristic of demand only, telling nothing about the relationship of demand to supply; average vacancy duration is the component that measures market condition.

In 1975, the national average vacancy duration was 6.3 weeks for owner units and 6.1 weeks for renter units, while the vacancy rate was 1.2 percent for owner units and 6.0 percent for renter units. The vacancy duration measure shows that average conditions in the two markets were essentially the same in 1975. If we had used the vacancy rate measure, we would have concluded that the owner markets were far tighter than the renter markets, when the real cause of different vacancy rates was the difference in annual turnover rate: 9.9 per 100 owner units and 51.2 per 100 rental units.

Applying the vacancy duration measure to the HASE sites shows that market conditions varied greatly by location (central South Bend, the rest of St. Joseph County, and Brown County) but only slightly by

type of housing (owner, single-unit rental, and multiunit rental). St. Joseph County had a loose housing market (12.7 weeks average vacancy duration in central South Bend and 8.8 weeks in the rest of St. Joseph County), and Brown County had a tight market (4.2 weeks average vacancy duration). The large difference, bracketing the national average vacancy duration of 6.2 weeks, shows that the experimental sites are good places to test the effects of market conditions on the housing allowance program.

One way of estimating average vacancy duration is to decompose the vacancy rate into turnover rate and vacancy duration: Divide the vacancy rate by the turnover rate. Neither the numerator nor the denominator is without estimation problems, however, so alternative methods of estimating average vacancy duration would be useful.

In principle, surveys of housing vacancies that ask "How long has this vacancy existed?" could provide a more direct method of estimating average vacancy duration. However, the answer to that question is not the *complete* duration of a vacancy but its duration up to the time of the survey (*interrupted* duration). Obviously, the average interrupted vacancy duration is not necessarily the same as the average complete vacancy duration. Not so obviously, the relationship between the two averages depends on the variance of the distribution of complete vacancy durations. The latter fact makes the supposedly more direct way of measuring average vacancy duration surprisingly indirect.

Information on variation in vacancy durations sheds considerable light on the nature of housing submarkets. That variation is high enough to prove that submarkets exist and that market conditions varied substantially across U.S. submarkets in 1975. The variation is far greater than can be explained by the variation of average vacancy duration among four regions and two types of housing. This note does not identify the submarkets, but it does show that they exist and are important.

CONTENTS

FIGURES

TABLES

I. INTRODUCTION

The traditional measure of housing market condition is the vacancy rate, the fraction of housing units that are vacant on an average day. A recent newsletter provides an example: "national data on vacancy rates continue to indicate a significant degree of tightness in the multi-family rental market with the rental vacancy rate nationally at only 5.4 percent in the third quarter of the year...."* New York City's rent-control law provides another example. Since 1964 that law has been contingent on the city council's determination that a public emergency exists. A vacancy rate of less than 5 percent is the criterion for such an emergency. (In 1965 the vacancy rate for New York City's rent-controlled units was 3.2 percent.)**

This working note proposes another measure of housing market condition: the annual average duration of a housing vacancy.*** It shows that annual average vacancy duration equals the vacancy rate divided by the annual turnover rate (see Sec. II). In other words, annual average vacancy duration is the vacancy rate normalized by the annual turnover rate.

If the turnover rate is stable, as in most longitudinal analyses of a given housing market, vacancy rate and vacancy duration are equivalent measures of housing market condition. However, if the turnover rate varies, as it does in cross-sectional analyses of

---

* *Economic Briefs*, Federal Home Loan Bank Board, Washington, D.C., November 1977, p. 4.

** Michael B. Teitz, *Rental Housing in New York City: Rent Control*, The New York City-Rand Institute and McKinsey & Co., RM-6316-1-NYC, May 1970, pp. 2-3.

*** We use the Bureau of the Census's definition of a vacancy, an unoccupied unit that is available for sale or rent. Accordingly, vacancy duration starts when a unit becomes vacant (either because an occupant moves out of an existing unit, or because the unit enters the market) and ends when it ceases to be for sale or rent (either because someone buys or rents the unit or because it is removed from the market).

different markets and submarkets, average vacancy duration is a different measure of market condition.

To illustrate how the two measures can differ, let us return to the newsletter's statement that a 5.4 percent vacancy rate indicates a "significant degree of tightness," and to the threshold vacancy rate of 5 percent for determining a housing emergency in New York City. The apparent agreement on what constitutes a tight housing market vanishes when we add the facts that the national average turnover rate for multifamily rental housing is about 60 turnovers per 100 housing units per year, while the turnover rate for New York City rent-controlled housing is about 20 turnovers per 100 units per year.*

Normalizing by the different turnover rates, the newsletter's 5.4 percent vacancy rate becomes an average vacancy duration of 4.7 weeks, and the rent-control law's 5 percent rate, a duration of 13.0 weeks.** Therefore, from the viewpoint of vacancy duration, the newsletter and the rent control law have very different conceptions of a tight housing market. Perhaps the rent-control benchmark was established using national averages without taking account of the differing turnover rates.

To choose between the competing measures we need a sharp definition of market condition. Intuitively, a tight market has excess demand and a loose market has excess supply. Market theory tells us that excess demand increases sales price and excess supply lowers sales price. Accordingly, we use the operational definition that one market is tighter than another if and only if equivalent housing has a higher market value in the first market than in the second. By equivalent housing we mean that both internal characteristics (e.g.,

---

*See Tables 2.1 and 3.1 for average turnover rates of rental housing. See Teitz, *Rental Housing in New York City: Rent Control*, pp. 38-39, for the turnover rates in New York City rent-controlled housing. In 1965, 16 percent of the occupants of rent-controlled units moved at least once during the year. Allowing for multiple moves, the estimated turnover rate was 20 per 100 units.

**For the newsletter, (5.4/60) = .09 years average vacancy duration, or 4.7 weeks. For New York City, (5.0/20) = .25 years average vacancy duration, or 13.0 weeks.

unit size and age) and external characteristics (e.g., lot size and neighborhood amenities) are the same.

That definition adequately incorporates the intuitive idea of market tightness. To the extent possible, owners will move housing capital from loose markets to tight markets to increase the market value of the capital. In the short run, of course, housing supply is immobile. The inability of supply to adjust as fast as demand can change is what causes housing market conditions to vary. In the long run, however, by not replacing deteriorated or accidentally destroyed housing in loose markets and by building new housing in tight markets, owners will shift housing capital.

Although we expect housing capital to shift from loose to tight markets, the converse is not necessarily true. The market with the larger net increase in housing stock is not necessarily the tighter market, because two markets can have different trends in aggregate demand. Assume that one market's supply is increasing to accommodate slowly increasing demand, and another market's supply is decreasing to accommodate slowly decreasing demand. Provided the demand changes are slow enough, the market conditions could be the same even though the net changes in housing stock are different.

In other words, the neutral benchmark for market condition can be either a static or a dynamic market equilibrium. Only under a static equilibrium (supply adjusted to a fixed aggregate demand function) will the net change in housing stock be zero. A dynamic equilibrium occurs when the aggregate demand function is changing but changing slowly enough that normal removals or feasible rates of new construction enable supply to stay adjusted to demand.

If the housing market is in static or dynamic equilibrium, the market value of housing equals its replacement cost (construction of improvements plus purchase of land at a price that capitalizes neighborhood amenities). In a loose market, housing value is less than replacement cost, stimulating supply to shrink to the equilibrium level (or time path). In a tight market, market value is greater than replacement cost, stimulating supply to increase to the equilibrium level.

Neither vacancy rate nor vacancy duration can possibly measure market condition perfectly, because those indicators only reflect the current relationship of supply to demand. To take an extreme example, suppose two markets currently have the same vacancy rates and average vacancy durations, but it is known that in one market the aggregate demand function will hold steady while in the other out-migration will cut it in half within a year. Clearly, market values of equivalent units will be lower in the second market than in the first, in spite of the identical current vacancy rates and vacancy durations.

However, rapid shifts in housing demand are not usually foreseeable. Most participants in the market probably take their cues from current experience. If so, the current relationship between supply and demand is sufficient to measure housing market condition, so the vacancy rate and the average vacancy duration become credible measures. The question is, Which one is better?

Evidence from the Housing Assistance Supply Experiment suggests that vacancy duration does a better job of measuring market condition than does vacancy rate. The first two columns of Table 1.1 show vacancy rate and vacancy duration for three types of housing in three locations in the HASE sites (central South Bend, the rest of St. Joseph County, Indiana, and Brown County, Wisconsin).

Those vacancy rates and vacancy durations are not perfectly correlated. The vacancy rates vary widely within each location, being low for owner units, higher for renter units on single-unit properties, and higher still for renter units on multiunit properties. Vacancy durations, however, vary little within each location and are highest for owner units (see Sec. III).

The last two columns of the table report estimates of market value. The first of them shows simply the average values of housing units of each type. The values reflect both the average quantity of housing capital per unit and its price, and quantities vary among the types in the table. To convert the market values to true prices, we use evidence presented elsewhere that the Brown County housing

Table 1.1

VACANCY RATE, VACANCY DURATION, AND MARKET VALUE BY TYPE OF RESIDENTIAL PROPERTY: ST. JOSEPH COUNTY, 1974, AND BROWN COUNTY, 1973

Type of Property	Average Vacancy Rate (%)	Average Vacancy Duration (weeks)	Market Value (1974 $) per Average Unit	Market Value (1974 $) per Unit of Housing Capital[a]
Central South Bend				
Owner	4.2	21.2	10,900	4,700
Renter, single-unit	10.4	9.8	7,500	6,400
Renter, multiunit	12.7	10.5	5,400	5,400
Rest of St. Joseph County				
Owner	1.9	9.6	22,100	9,500
Renter, single-unit	7.3	8.2	10,300	8,800
Renter, multiunit	9.3	8.0	9,400	9,400
Brown County				
Owner	0.8	5.2	26,200	11,300
Renter, single-unit	4.4	4.5	13,200	11,300
Renter, multiunit	5.2	3.9	11,300	11,300

SOURCE: Vacancy rate and duration from Table 3.1. Market value from owner estimates of property value, obtained in HASE baseline surveys. Brown County data for 1973 are adjusted for price inflation in 1973-74.

[a]The unit of housing capital is an apartment on a multiunit property. Owner units have 2.32 units of housing capital per average housing unit, and renter units on single-unit properties have 1.17 units of housing capital per average housing unit. The adjustment factors were chosen to make the market value per unit of housing capital constant in Brown County because we judge that market to be in equilibrium.

market is in equilibrium.* Accordingly, the price of a unit of housing capital is constant across all types of housing in Brown County, and the market values per average unit show how the quantity of housing per unit varies by type of housing. We conclude that the average owner unit is equivalent to 2.3 apartments on multiunit properties, and that a renter unit on a single-unit property is equivalent to 1.2 apartments on multiunit properties.

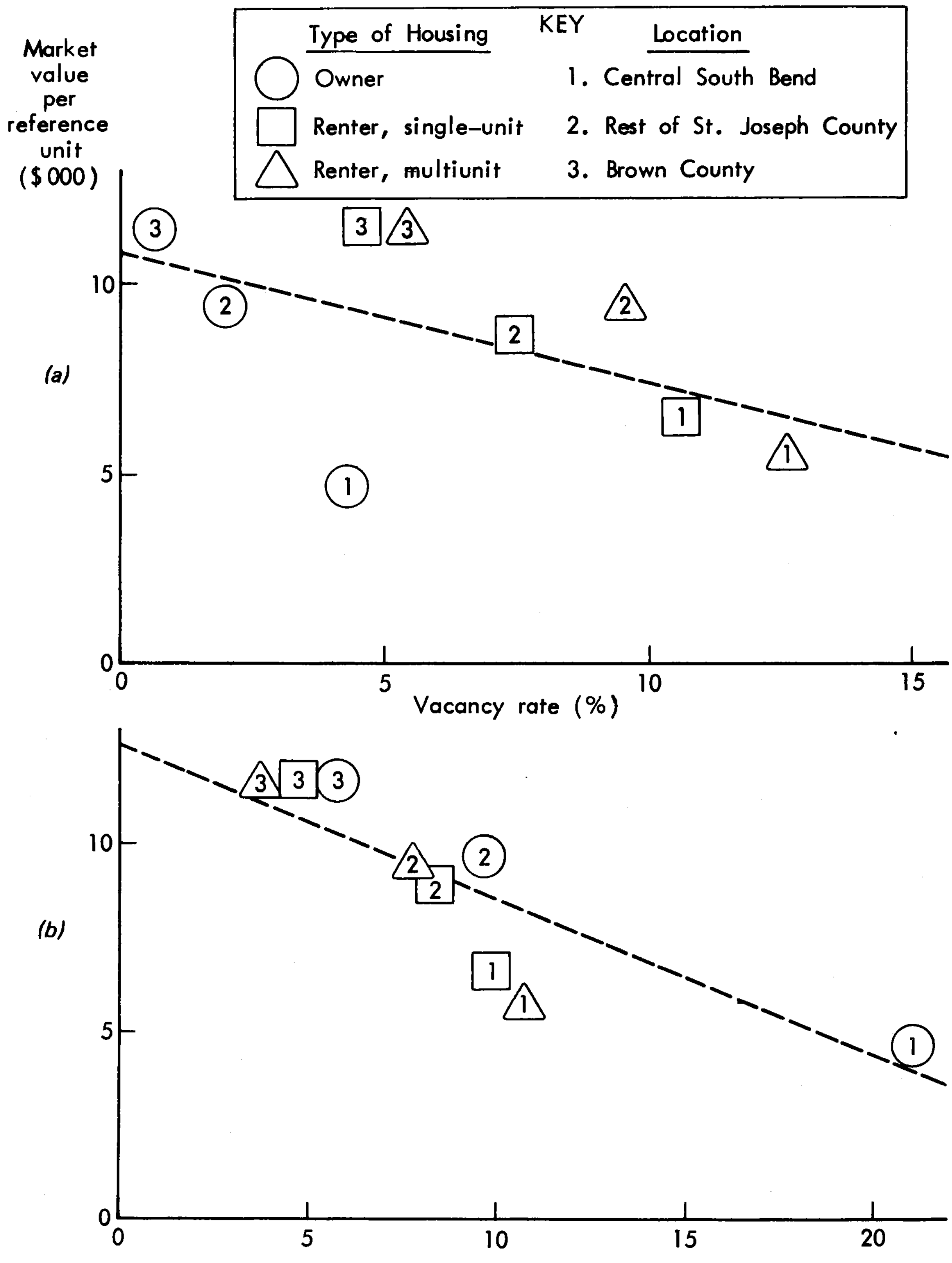
Applying those adjustment factors in all three locations produces the fourth column of Table 1.1, market value per unit of capital. Those values provide an operational definition of market condition: The greater the market value per unit of capital, the tighter the housing market.

Graphing vacancy rate and vacancy duration against market value per unit of capital, we find that vacancy rate does a poor job of predicting market condition (see Fig. 1*a*), while vacancy duration does a much better job (see Fig. 1*b*). The dotted lines in the figures show the linear regressions of market value on each indicator. Vacancy duration explains 71 percent of the variation in market values; vacancy rate explains only 31 percent. The vacancy duration regression is significant at the 99 percent confidence level, but the vacancy rate regression is not significant even at the 95 percent confidence level.

National data also support the conclusion that vacancy duration predicts market condition better than vacancy rate. Because housing tenure can be changed relatively easily, it is reasonable to assume that owner-occupied housing and renter housing have about the same

---

*In Brown County the real values of rental property did not change significantly from 1960 to 1973, indicating a dynamic equilibrium in the housing market--a running balance between supply and demand even though both were changing. In contrast, the real values of rental housing fell by 24 percent in central South Bend and by 4 percent in the rest of St. Joseph County from 1961 to 1974. The decline in value reflects the county's population losses during the period, which left a price-depressing surplus of housing, especially in the urban core. See *Third Annual Report of the Housing Assistance Supply Experiment*, The Rand Corporation, R-2151-HUD, February 1977, pp. 67-70.



SOURCE: Table 1.1

Fig. 1—Relationship of market value *(a)* to vacancy rate and *(b)* to vacancy duration: nine housing submarkets

market conditions--at least when many housing markets are averaged. However, Sec. II shows that the national average vacancy rates for owner and renter units are not even close (1.2 percent vs. 6.0 percent, in 1975), while the national average vacancy durations for owner and renter units are almost identical (6.3 weeks vs. 6.1 weeks, in 1975).

Empirical regularities always beg for theoretical explanations. Unfortunately, we do not have a housing market theory that explains why average vacancy duration predicts market condition better than vacancy rate. A recent theoretical paper recognizes that vacancy rate and vacancy duration are different indicators and argues that both are necessary to understand market condition.* However, it does not identify the circumstances under which vacancy duration would necessarily be a better measure than vacancy rate.

To inform both empirical and theoretical analyses of housing market condition, this note estimates average vacancy durations nationally, regionally, and locally in Secs. II and III. Appendix A gives the methodology for the estimates.

Then, in Sec. IV, average *complete* vacancy duration is contrasted with average *interrupted* vacancy duration. The latter is the average answer to the vacancy survey question, "How long has this vacancy existed?" It turns out that average interrupted vacancy duration depends as much on the variance of the vacancy duration distribution as on its central tendency. In Sec. V, we use the information on variation in vacancy durations to better understand the structure of housing submarkets.

---

*Chang-i Hua, *The Equilibrium of Housing Vacancy and Waiting Time*, School of Urban and Public Affairs, Carnegie-Mellon University, Pittsburgh, Pennsylvania, September 1977, pp. 12-13.

II. VACANCY DURATION VS. VACANCY RATE

This section proves that annual average vacancy duration equals the annual average vacancy rate divided by the annual average turnover rate. In other words, the vacancy rate is the product of the turnover rate and average vacancy duration. Average vacancy duration is the more useful measure of market condition, but the vacancy rate is easier to estimate and is equivalent to vacancy duration for comparing markets with identical turnover rates. However, we show that turnover rates vary dramatically among housing markets, making it preferable to use average vacancy duration for cross-sectional analyses of housing market conditions.

If the same number of turnovers occurs each day, and if every vacancy created by a turnover lasts the same number of days, it is obvious that the vacancy duration equals the vacancy rate divided by the turnover rate. For example, if 3 percent of the housing units have turnovers every day and if each vacancy lasts two days, there will be 6 percent vacancies on any given day (half starting the previous day and half starting on the day in question). The following proof shows that the relationship also occurs when the turnover rate varies by season and when vacancies have varying durations.

The rigorous proof of the relationship between vacancy duration and vacancy rate depends on an assumption that the turnover process is "annually cyclical," that even though the rate of turnover and the distribution of vacancy durations change during the year, the annual pattern is the same each year. If that assumption is only approximately true, the average vacancy duration only approximately equals the ratio of vacancy rate to turnover rate. As a practical matter, only very large deviations from annual cyclicity would significantly disturb the stated relationship.

The arithmetic is simplest if the same unit of time is used to scale vacancy duration and turnover. In the proof below we use days as the unit of time; later, it will be convenient to describe results per week or per year.

By definition, the annual average duration of a vacancy is the weighted average of average vacancy durations beginning in each part of the year, the weights being the turnover distribution:

$$E(x) = \sum\left(\frac{T_k}{\Sigma T_k}\right) E_k(x) = \frac{\Sigma\ T_k E_k(x)}{\Sigma\ T_k}, \tag{1}$$

where k = day of year (and the summations are over k = 1 to 365),

x = duration of a vacancy (in days),

T_k = turnover on day k, i.e., vacancies started on day k,

$E_k(x)$ = average duration of vacancies started on day k, and

$E(x)$ = average duration of vacancies started during a year.

The denominator of the right-hand ratio in Eq. (1) is the total number of turnovers during a year, and the numerator is the total number of vacant unit-days generated by the year's turnovers. Some of the vacant unit-days generated by a year's turnovers occur in the following year. However, assuming a fixed annual turnover cycle, that loss is exactly balanced by the vacant unit-days in the current year that were generated by the previous year's turnovers. Therefore, the total vacant unit-days generated by a year's turnover equals the total vacant unit-days in a year:

$$\Sigma\ T_k E_k(x) = \Sigma\ V_k, \tag{2}$$

where V_k = vacant units on day k.

Putting Eq. (2) into Eq. (1) gives us vacancies in the numerator and turnovers in the denominator. Dividing both numerator and denominator by total unit-days, we conclude that the annual average vacancy duration (in days) equals the ratio of the annual average vacancy rate to the annual average turnover rate (in turnovers per unit per day):

$$E(x) = \frac{\Sigma V_k}{\Sigma T_k} = \frac{\Sigma V_k/\Sigma H_k}{\Sigma T_k/\Sigma H_k} = \frac{\sum\left(\frac{H_k}{\Sigma H_k}\right)\left(\frac{V_k}{H_k}\right)}{\sum\left(\frac{H_k}{\Sigma H_k}\right)\left(\frac{T_k}{H_k}\right)}, \tag{3}$$

where H_k = housing units in market on day k.

If the number of housing units in a market is constant during the year, so that $H_k = H$ for all k, and $\Sigma H_k = 365H$, then the formula for annual average vacancy duration, $E(x)$, simplifies to:

$$E(x) = \frac{\frac{1}{365}\sum_{k=1}^{365}\left(\frac{V_k}{H}\right)}{\frac{1}{365}\sum_{k=1}^{365}\left(\frac{T_k}{H}\right)}, \tag{4}$$

which makes the annual averages of vacancy rate and turnover rate very clear. However, the more complex averages in Eq. (3) are useful because they show that the relationship does not require the number of housing units to be constant during the year.

We have proved that average vacancy duration equals the vacancy rate divided by the annual turnover rate. Now we show that average vacancy duration and average vacancy rate are not always equivalent measures of housing market condition because the turnover rate can vary cross-sectionally.

The first two columns of Table 2.1 give the vacancy rate and annual turnover rate by region and tenure for the United States in 1975. The third column estimates average vacancy duration as the ratio of the vacancy rate to the turnover rate (multiplying the ratio by 52 to convert the answer to weeks). For example, the national average vacancy duration is 6.2 weeks--the ratio of the average vacancy rate of 3.0 percent and the turnover rate of 25.1 turnovers per 100 units per year.

In all four regions, owner units turn over less rapidly than renter units. The national averages are 9.9 and 51.5 turnovers per

Table 2.1

VACANCY RATE, TURNOVER RATE, AND VACANCY DURATION BY TENURE AND REGION: UNITED STATES, 1975

Tenure	Average Vacancy Rate (%)	Annual Turnover per 100 Units	Average Vacancy Duration (weeks)[a]
Northeast			
Owner	1.0 (.1)	6.0 (.3)	8.7 (1.0)
Renter	4.1 (.2)	36.5 (.8)	5.8 (.3)
All	2.3 (.1)	18.7 (.4)	6.5 (.3)
North Central			
Owner	1.0 (.1)	9.5 (.3)	5.5 (.5)
Renter	5.7 (.2)	52.0 (1.1)	5.7 (.2)
All	2.5 (.1)	23.0 (.4)	5.6 (.2)
South			
Owner	1.5 (.1)	11.0 (.3)	7.1 (.5)
Renter	7.7 (.2)	57.2 (1.0)	7.0 (.2)
All	3.7 (.1)	27.0 (.4)	7.1 (.2)
West			
Owner	1.5 (.1)	13.1 (.5)	6.0 (.5)
Renter	6.2 (.2)	61.6 (1.2)	5.2 (.2)
All	3.4 (.1)	32.4 (.6)	5.4 (.2)
All United States			
Owner	1.2 (.05)	9.9 (.2)	6.3 (.3)
Renter	6.0 (.1)	51.5 (.5)	6.1 (.1)
All	3.0 (.05)	25.1 (.2)	6.2 (.1)

SOURCE: Vacancy rate from *Housing Vacancies*, 1975, Bureau of the Census, Series H-111-75-5, Table 1, p. 12. Turnover rate estimated by method described in Appendix A from data in *Annual Housing Survey: 1975, Part A*, Bureau of the Census, Series H-150-75A, Tables 1 and 5 in Secs. A through E.

NOTE: Numbers in parentheses are standard errors of estimate (see Appendix A for their derivation).

[a]Estimated by 52 times the ratio of vacancy rate to annual turnover rate.

100 units per year. The differences in turnover rates are so large that even though the vacancy rates for owner units are only about one-fifth those for renter units, the average vacancy duration for owner and renter units is essentially the same in three out of four regions. In the fourth region, the Northeast, owner units are vacant 1.5 times as long as renter units (8.7 weeks vs. 5.8 weeks).*

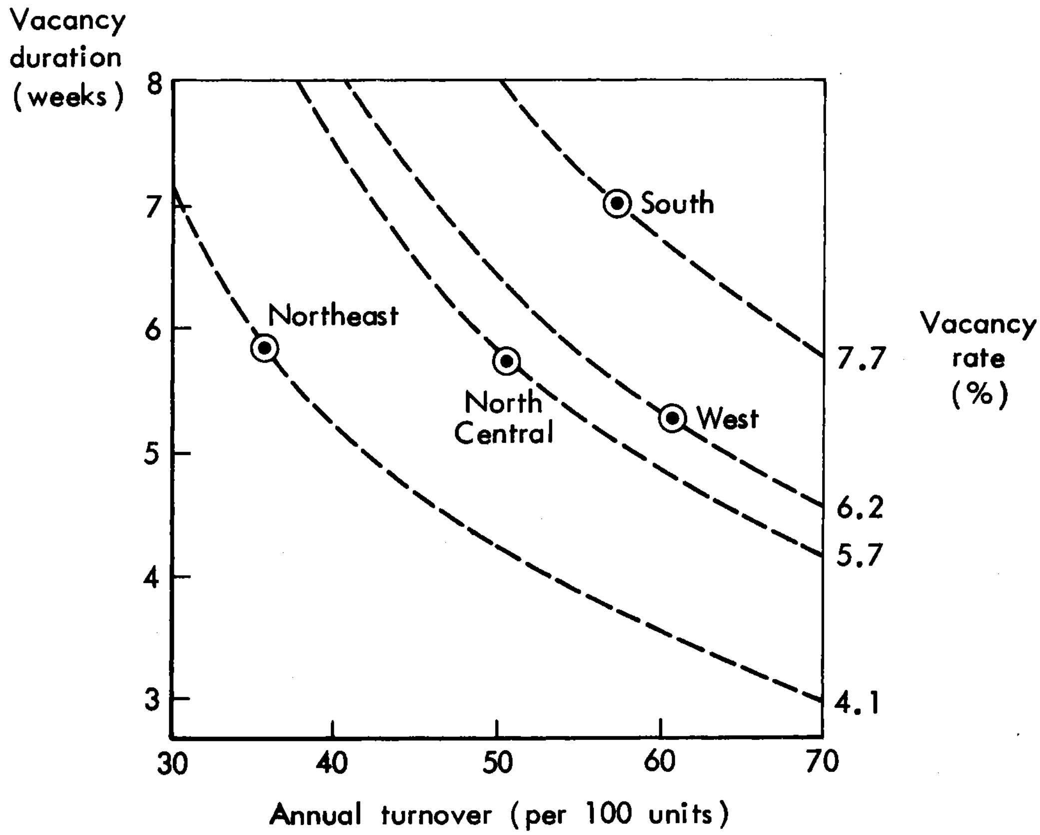
The regional pattern in turnover rates is the same for both owner and renter units: the Northeast region has the lowest rate and the North Central, South, and West regions have progressively higher ones. The turnover rates in the West are more than double those in the Northeast for owner units (13.1 vs. 6.0) and two-thirds larger for renter units (61.6 vs. 36.5). The most probable explanation for the pattern is that the nation's more mobile households have been moving from the Northeast to the West in recent decades, reducing the average turnover rate in the population left behind.

Whatever their cause, the regional differences in turnover rates mean that vacancy rates and vacancy durations measure regional housing market conditions differently. If we used the vacancy rate as a measure, we would conclude that the Northeast has the tightest housing market even though it has been losing population to the other regions. However, the low vacancy rates in the Northeast are caused not by excess demand but by low turnover. Looking at average vacancy duration, we see that the Northeast's ownership market is considerably looser than the national average and its renter market is only slightly tighter than the national average.

Focusing on the rental markets, Fig. 2 shows graphically how vacancy duration ranks regional markets differently than do vacancy rates. The vertical axis is vacancy duration; the horizontal axis is annual turnover. The dotted lines connect combinations of vacancy duration and turnover that result in the same vacancy rate.

---

*The difference is significantly different from zero at the 95 percent confidence level because the interval estimate of the difference is 2.90 ± 2.04 weeks, which does not include zero. The half-interval was estimated using the standard errors given in parentheses in Table 2.1: $(1.96)\sqrt{(1.0)^2 + (.3)^2} = 2.04$.



SOURCE: Table 2.1

Fig. 2—Vacancy rate, turnover rate, and average vacancy duration by region: rental housing, United States, 1975

While both vacancy duration and vacancy rate indicate that the South has the loosest rental market, the two measures rank the remaining three regions inversely. The Northeast has the lowest vacancy rate, but because its turnover rate is also lowest its average vacancy duration is about the same as those in the North Central and West.

III. APPLICATION TO THE HOUSING ASSISTANCE SUPPLY EXPERIMENT

The Housing Assistance Supply Experiment is evaluating the effects of a full-scale housing allowance program on the participants and local markets in Brown County, Wisconsin (whose central city is Green Bay) and St. Joseph County, Indiana (whose central city is South Bend). Those sites were chosen for their contrasting market conditions, so that the effects of market conditions on allowance program impacts could be tested.* This section uses the average vacancy duration measure to show the extent to which market conditions in the sites differed at the start of the experiment (1973 in Brown County and 1974 in St. Joseph County).

St. Joseph County has a loose housing market, with an average vacancy duration of 12.7 weeks in central South Bend** and 8.8 weeks in the rest of the county. Brown County has a tight housing market, with an average vacancy duration of only 4.2 weeks (see Table 3.1). Those vacancy durations bracket both the national average of 6.2 weeks and the North Central regional average of 5.6 weeks. Therefore, the experimental sites enable good tests of the effects of market conditions.***

The ranking of housing market conditions by experimental location is the same whether vacancy rate or vacancy duration is the measure of market condition. Both measures show that the market in central

---

*For an overview of HASE objectives, see *Third Annual Report of the Housing Assistance Supply Experiment*, pp. 2-4.

**Central South Bend includes all but the fringes of the city of South Bend and has about half the rental units and one-fourth the owner units in St. Joseph County.

***So far the experiment has shown that the housing allowance program does *not* cause rent increases in either tight or loose housing markets and that rents for equivalent housing vary little with market condition. See C. Peter Rydell, *Effects of Market Conditions on Prices and Profits of Rental Housing*, The Rand Corporation, P-6008, September 1977, and *Third Annual Report of the Housing Assistance Supply Experiment*.

Table 3.1

VACANCY RATE, TURNOVER RATE, AND VACANCY DURATION BY TYPE OF RESIDENTIAL PROPERTY: ST. JOSEPH COUNTY, 1974, AND BROWN COUNTY, 1973

Type of Property	Average Vacancy Rate[a] (%)	Annual Turnover per 100 Units[b]	Average Vacancy Duration (weeks)[c]
Central South Bend			
Owner	4.2 (1.1)	10.3 (2.5)	21.2 (9.3)
Renter, single-unit	10.4 (.6)	55.4 (4.3)	9.8 (.9)
Renter, multiunit	12.7 (.5)	62.6 (3.7)	10.5 (.7)
All	7.3 (.7)	29.8 (1.9)	12.7 (1.4)
Rest of St. Joseph County			
Owner	1.9 (.5)	10.3 (1.5)	9.6 (2.8)
Renter, single-unit	7.3 (.5)	46.5 (3.6)	8.2 (.8)
Renter, multiunit	9.3 (.5)	60.5 (2.4)	8.0 (.5)
All	3.0 (.4)	17.8 (1.3)	8.8 (1.3)
Brown County			
Owner	0.8 (.3)	8.0 (.9)	5.2 (.6)
Renter, single-unit	4.4 (.3)	51.1 (2.6)	4.5 (.4)
Renter, multiunit	5.2 (.2)	68.6 (1.8)	3.9 (.2)
All	2.1 (.2)	26.0 (.8)	4.2 (.4)

SOURCE: HASE baseline surveys of tenants, homeowners, and landlords (excluding mobile home, rooming house, farm, and federally subsidized properties).

NOTE: Numbers in parentheses are standard errors of estimate (see Appendix A for their derivation).

[a]Owner vacancy rate estimated by percent of units vacant at time of survey. Renter vacancy rate estimated by the annual percent of rent lost because of vacancies (adjusted for variation in the rent-loss rate by rent strata).

[b]Estimated by dividing all move-ins to rental units (as reported in the tenant survey's mobility histories) by rental units. The move-in rate measures turnover because the baseline study population is limited to properties that existed all year.

[c]Estimated by 52 times the ratio of vacancy rate to annual turnover rate.

South Bend is looser than that in the rest of St. Joseph County, and that the latter is looser than the market in Brown County.* The two measures are equivalent in this instance because the average turnover rate does not vary enough by experimental location to drive the two measures in different directions.

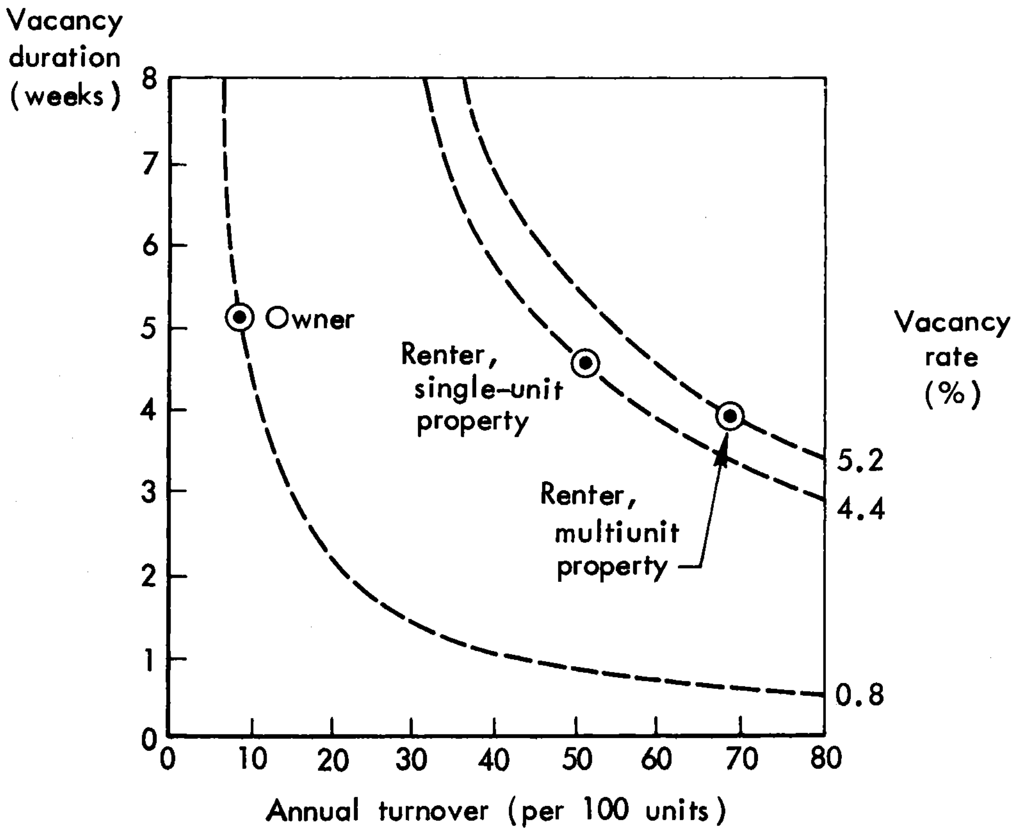
However, the ranking of housing market conditions by type of property within location does depend on which measure is used. If we used the vacancy rate measure we would conclude that market conditions vary even more by type of property than by location, with owner units having the tightest markets and renter units on multiunit properties having the loosest markets in all three locations. Instead, using the vacancy duration measure we conclude that market conditions vary less by type of property than by location, and that owner units have looser markets than renter units (especially in central South Bend).

The reason the two measures rank market conditions differently is that the average turnover rate varies systematically by type of property. Owner units have less turnover than renter units, and renter units on single-unit properties have less turnover than renter units on multiunit properties. Table 3.1 shows that pattern in all three locations.

In Brown County, for example, the vacancy rate varies from 0.8 percent for owner units to 5.2 percent for renter units on multiunit properties. But the turnover rate varies in the same way, and to an even greater extent, making the ratio of vacancy rate to turnover rate, i.e., the average vacancy duration, larger for owner units than for renter units on multiunit properties (see Fig. 3).

---

*The standard errors of estimate (the numbers in parentheses in Table 3.1) are large enough so that detailed t-tests had to be performed to reach this conclusion. The only exceptions occur in some details of the comparison between central South Bend and the rest of St. Joseph County. The vacancy rates of owner units and the vacancy durations for single-unit rental properties are not significantly different at the 95 percent confidence level. However, they are significantly different at the 68 percent confidence level (the 84 percent confidence level by a one-tailed test).



SOURCE: Table 3.1

Fig. 3—Vacancy rate, turnover rate, and average vacancy duration by type of residential property: Brown County, Wisconsin, 1973

The reversal of ratings in Fig. 3 shows that vacancy duration is a different measure of market conditions than vacancy rate. The figure also shows that in Brown County vacancy rates vary greatly by property type while vacancy durations vary little. Brown County has had moderate, predictable growth in housing demand over the past two decades. Its housing market seems to be in dynamic equilibrium, i.e., supply tracking demand without large surpluses or shortages. It is not surprising that its submarkets are uniformly tight, as the average vacancy duration measure shows.

St. Joseph County, in contrast, has lost population (especially in Central South Bend) during the past two decades. It has an excess supply of housing, the amount varying by location and by type of property. Table 3.1 shows average vacancy durations that vary from 21.2 weeks for owner units in central South Bend to 8.0 weeks for multiunit rental properties in the rest of St. Joseph County.

IV. COMPLETE VS. INTERRUPTED VACANCY DURATION

Estimating an average vacancy duration* as the ratio of a vacancy rate to a turnover rate, while theoretically sound, is not always empirically easy. Both vacancy rate and turnover rate can be difficult to measure (see Appendix A). It would be useful to have a more direct way to measure vacancy duration.

The following argument shows the extent to which average interrupted vacancy duration (time from the start of a vacancy to the date of a vacancy survey) can be used to estimate average vacancy duration (time from the start of a vacancy to its end). It turns out that such estimation is possible only if we know the coefficient of variation for vacancy durations. Lacking that information, we cannot use average interrupted vacancy duration to estimate average vacancy duration. However, if we can measure both interrupted and complete vacancy durations, we can use those data to estimate the coefficient of variation for vacancy durations.

Housing vacancy surveys conducted by the U.S. Census ask how long a vacancy has existed. Even though the answer to that question only gives the duration up to the time of the survey, it seems intuitively plausible that the answer would be strongly related to the average vacancy duration.

Reasoning that, on average, a vacancy survey interrupts a vacancy at its midpoint, we might guess that "interruption bias" makes the average answer to the census question equal half the average vacancy duration. However, that guess would be dramatically wrong. The average interrupted vacancy duration is several times greater than the average vacancy duration: 4.3 times greater for owner units and 2.7 times greater for renter units (see Table 4.1).

---

*Wherever it appears in this report, "average vacancy duration" refers to the total time between the start of a vacancy and its end. This section contrasts that *complete* duration with the *interrupted* duration measured in vacancy surveys; for simplicity, the word "complete" is usually omitted.

Table 4.1

VACANCY DURATION AND INTERRUPTED VACANCY DURATION: UNITED STATES, 1975

Tenure	Average Vacancy Duration[a] (weeks)	Average Interrupted Vacancy Duration[b] (weeks)	Ratio of Interrupted to Complete Durations
Owner units	6.3 (.3)	27.4 (.7)	4.3 (.23)
Renter units	6.1 (.1)	16.3 (.4)	2.7 (.08)
All units	6.2 (.1)	19.2 (.4)	3.1 (.08)

SOURCE: Tables 2.1 and B.1.

NOTE: Numbers in parentheses are standard errors of estimate (see Appendixes A and B for their derivation).

[a] Average duration of all the vacancies generated by a year's turnover, as estimated by the ratio of vacancy rate to turnover rate.

[b] Average answer to the vacancy survey question: "How long has this unit been vacant?"

How can the part be larger than the whole? The paradox results from "duration bias" in the answers to the census question. Longer vacancies have more chance of encompassing the survey date, so are weighted more heavily in the average interrupted vacancy duration than in the average vacancy duration. The greater the variation in vacancy durations, the greater the "duration bias" and the greater the ratio of interrupted to complete durations.

The combined effects of interruption bias and duration bias can be stated precisely:

$$\frac{E(i)}{E(x)} = \frac{(1 + C^2)}{2}, \tag{5}$$

where $E(i)$ = average interrupted vacancy duration,

$E(x)$ = average (complete) vacancy duration, and

C = coefficient of variation in vacancy durations, $\sigma_x/E(x)$, where σ_x is the standard deviation.

If the coefficient of variation is zero, i.e., if all vacancy durations are the same, then there is pure interruption bias and the average interrupted vacancy duration equals one-half the average vacancy duration. If the coefficient of variation is 1.0, then duration bias exactly counteracts interruption bias and the two average durations are equal. In the turnover processes we observe nationally, the coefficient of variation must be greater than 1.0 because the average interrupted vacancy duration exceeds the average vacancy duration. Table 4.2 gives the precise coefficients of variation implied by the data in Table 4.1.

Table 4.2

COEFFICIENT OF VARIATION
FOR VACANCY DURATIONS

Tenure	Coefficient of Variation[a]
Owner units	2.8 (.14)
Renter units	2.1 (.06)
All units	2.3 (.05)

SOURCE: Equation (5) and Table 4.1.
NOTE: Numbers in parentheses are standard errors of estimate (see Appendix B).

[a]The standard deviation (square root of the variance) divided by the mean.

To prove Eq. (5), we analyze the effects of interruption bias and duration bias separately and then combine the results. Both parts of the proof require the assumption that the turnover process is annually cyclical.*

---

*The proof follows a similar theory based on the duration of unemployment in Stephen W. Salant, "Search Theory and Duration Data: A Theory of Sorts," *Quarterly Journal of Economics*, Vol. 91, No. 1, February 1977, pp. 39-41, except that Salant assumes a stationary instead of an annually cyclical process.

If the question, "How long has this unit been vacant?" is asked about all vacant units on a given day, every day for a year, and then if all answers are averaged, the result is annual average interrupted vacancy duration.

For a given vacancy the interrupted duration is less than or equal to the complete duration. Interruption bias tends to make the average interrupted duration less than the average complete duration. In fact, if all vacancies had the same duration, then the average interruption duration would be half the complete duration:*

$$E(i|x) = \frac{x}{2} , \tag{6}$$

where x = duration of a vacancy,

i = interrupted duration of a vacancy, and

$E(i|x)$ = expected interrupted duration, i, given the complete duration, x.

However, not all vacancies have the same duration, and longer vacancies are observed more days (i.e., weighted more heavily) in the computation of annual average interrupted duration. Specifically, the average frequency with which a daily vacancy survey would observe vacancies that last x days is proportional to x as well as to the fraction of all vacancies generated during a year that are x days long:**

---

*To derive Eq. (6) we need to assume an annually cyclical turnover process. Observing an x-day vacancy on each day of its existence obviously leads to an average interrupted duration of $x/2$. The assumption of annually cyclical turnover makes observing all x-day vacancies on all days of a year the same as observing all x-day vacancies generated during a year on every day of their existence.

**To derive Eq. (7) we again need to assume an annually cyclical turnover process. Observing vacancies each day they exist obviously makes observations of x-day vacancies proportional to x as well as to the frequency of x-day vacancies. The assumption of an annually cyclical turnover makes observing all vacancies on all days of the year the same as observing all vacancies generated during a year on all the days they exist.

$$g(x) = \frac{x\ f(x)}{E(x)}\ , \tag{7}$$

where $f(x)$ = fraction of vacancies generated during a year that last x days.

$g(x)$ = fraction of observations during the year (making new observations each day) that are of vacancies lasting x days, and

$E(x) = \sum_x x\ f(x)$ = average duration of vacancies generated during a year.

Multiplying $E(i|x)$, the average interrupted vacancy duration, by $g(x)$, and summing over all complete durations, we find the desired relationship between the average interrupted vacancy duration and the average vacancy duration:*

$$E(i) = \sum_x E(i|x)g(x) = \frac{\sum_x x^2 f(x)}{2\ E(x)} = \frac{E(x)}{2}\ [1 + C^2], \tag{8}$$

where $E(i)$ = average interrupted vacancy duration,

$E(x)$ = average (complete) vacancy duration, and

C = coefficient of variation for vacancy durations.

To summarize, we looked at average interrupted vacancy duration in hopes of finding a measure of average vacancy duration that was more direct than the ratio of vacancy rate to turnover rate. Instead, we found that the average interrupted duration depends as much on the variance of the duration as on the mean, making the approach a remarkably indirect way of estimating average duration. However, the information on the variation of vacancy durations turns out to be valuable, as will be seen in the next section.

---

*The details of the derivation require $\Sigma x^2 f(x) = E(x^2)$, $var(x) = E(x^2) - E(x)^2$, and $C^2 = var(x)/E(x)^2$.

V. THE IMPORTANCE OF SUBMARKETS

Submarkets are parts of the housing market, delineated either by location or type of housing, that have low cross-elasticities of demand. Households seeking a dwelling in one submarket will, by definition, not readily accept one in another submarket. On the other hand, households consider all dwellings within a submarket to be substitutes.

Submarkets obviously exist because households vary in their preferences for locations and types of housing. The question is, Are submarkets different enough to be important? We have to settle for indirect evidence, because it is difficult to observe cross-elasticities of demand. We must look for a characteristic of housing markets that can manifest itself only if submarkets exist and are very different. A coefficient of variation for vacancy durations that is considerably larger than 1.0 is such a characteristic.

Although the existence of submarkets does not necessarily imply a high coefficient of variation for vacancy durations, this section shows that the coefficient cannot greatly exceed 1.0 in the absence of greatly differing submarkets. The evidence does not define the submarkets, but it confirms that they exist and are important.

We have just seen (Table 4.2) that the coefficient of variation for vacancy durations in the United States was 2.3 in 1975. Random variation in vacancy durations caused by uniform vacancy-ending rates cannot account for such a high coefficient. If all vacancies had the same daily probability of ending, then the coefficient of variation for vacancy duration would equal 1.0.*

Nor is it plausible that vacancy-ending rates that change during the course of a vacancy are the reason for the high variation in vacancy durations. If the daily probability of a vacancy-ending changes

---

*Uniform vacancy-ending rates cause an exponential distribution of vacancy durations, where the standard deviation equals the mean, so the coefficient of variation is 1.0 (see the benchmark case in Appendix C).

at all during a vacancy, it will presumably do so because landlords and owners become impatient and lower the rent or purchase price. Those actions would increase the vacancy-ending rate. However, Appendix C shows that vacancy-ending rates that increase over time cause the coefficient of variation for vacancy durations to be less than 1.0. So the hypothesis of nonconstant vacancy-ending rates cannot explain why the coefficient of variation exceeds 1.0.

The remaining possibility is that vacancy-ending rates, though constant for a given unit, are not uniform across units. Among vacancies that start at the same time, the ones with the low vacancy-ending rates would become an increasing proportion of the surviving vacancies. Therefore, the ending rate for the cohort's surviving vacancies would decrease over time. Appendix C shows that under those circumstances, the coefficient of variation for vacancy duration is greater than 1.0.

Thus, nonuniform vacancy-ending rates are the explanation for the high variation in vacancy durations. That implies that submarkets exist, because only submarkets can make vacancy-ending rates differ across housing units. If housing units were perfect substitutes, they would have the same probabilities of being sold or rented on a given day.

Within a submarket, units *are* perfect substitutes, so all vacancies have the same daily probability of ending. Vacancy durations thus are exponentially distributed, so that the expected vacancy duration in the submarket equals the inverse of the vacancy-ending rate, and the coefficient of variation for vacancy durations in the submarket is 1.0. Across submarkets with different market conditions, the vacancy-ending rates and expected vacancy durations would vary, so that the overall distribution of vacancy durations would have a coefficient of variation greater than 1.0.*

---

*The theorem in Appendix C shows that a distribution that is a mixture of exponential distributions has a coefficient of variation greater than 1.0.

We conclude that a coefficient of variation for vacancy durations greater than 1.0 shows that submarkets exist. The converse is not necessarily true, because submarkets might all have the same expected vacancy durations and vacancy-ending rates.

So far this argument has used only the fact that the observed coefficient of variation for vacancy durations is larger than 1.0 to show that submarkets exist. Because the coefficient is considerably larger than 1.0, we also conclude that submarket conditions in the United States during 1975 were more than trivially different. A coefficient of variation greater than 2.0 is very large. Expected vacancy duration has to vary considerably among submarkets to generate that much variation in realized vacancy durations.

The formula for computing the overall coefficient of variation for vacancy durations from submarket characteristics is

$$C = \sqrt{\frac{2\Sigma\ W_k[E_k(x)]^2}{[\Sigma\ W_k E_k(x)]^2} - 1}, \qquad (9)$$

where C = coefficient of variation for vacancy durations,

W_k = proportion of turnovers that occur in submarket k, and

$E_k(x)$ = expected vacancy duration in submarket k.*

The formula assumes that the expected vacancy duration, $E(x)$, is constant within a submarket during a year. However, Appendix A shows that average vacancy duration is about twice as long in winter (when turnover is low) as in summer (when turnover is high). Adding the

---

*In the kth submarket, vacancy durations are distributed exponentially, so that the first and second moments are $E_k(x)$ and $2[E_k(x)]^2$. The first and second moments of the distribution across all submarkets are just the weighted averages of the submarket moments, $\Sigma\ W_k E_k(x)$ and $2\Sigma\ W_k[E_k(x)]^2$, where the weights, W_k, are the proportion of turnovers occurring in each submarket ($\Sigma W_k = 1$). Equation (9) follows because the variance equals the second moment less the square of the first moment; the standard deviation is the square root of the variance; and the coefficient of variation is the standard deviation divided by the mean. Note that if all the $E_k(x)$ are the same, i.e., if there are no submarkets, then $C = 1$.

contribution of seasonal variation to the annual coefficient of variation produces:*

$$C = \sqrt{\frac{2.138\, \Sigma\, W_k [E_k(x)]^2}{[\Sigma\, W_k E_k(x)]^2} - 1}. \qquad (10)$$

In Table 5.1 Eq. (10) is used to compute the coefficient of variation for vacancy durations if the only U.S. submarkets were those defined by region and tenure. In other words, we assume that the annual expected duration of a vacancy is uniform within each of the eight region-tenure groups, observe that the expected duration varies between groups from 5.2 to 8.7 weeks, and calculate the corresponding coefficient of variation in realized vacancy durations. The answer is only 1.1.

We conclude that market conditions vary much more within the region-tenure groups than between them. To gauge how large the intragroup differences must be, Table 5.2 applies Eq. (10) to St. Joseph and Brown counties. We again assume uniform expected vacancy durations within each location-type group, but now the variation in expected duration is greater, ranging from 3.9 to 21.2 weeks. The resulting coefficient of variation is 1.3--still nowhere near 2.3.

Assuming that the HASE sites provide a cross-section of U.S. housing market conditions, we conclude that there must be housing submarkets even more detailed than those defined by the distinctions in Table 5.2. (The alternative possibility is that market conditions vary more dramatically among counties in the United States than between Brown and St. Joseph counties.)

---

*Within each submarket the distribution of annual turnover in the four quarters of the year is approximately .17, .25, .33, and .25. Accordingly, vacancy durations in the four quarters are 1.50 $E_k(x)$, 1.00 $E_k(x)$, 0.75 $E_k(x)$, and 1.00 $E_k(x)$, where $E_k(x)$ is the annual average vacancy duration for the submarket. Therefore, the sum of squared vacancy durations is $[.17(1.50)^2 + .25(1.00)^2 + .33(0.75)^2 + .25(1.00)^2]$ $[E_k(x)]^2$, which equals $1.069[E_k(x)]^2$. Using that factor in place of the $[E_k(x)]^2$ factors in Eq. (9) produces Eq. (10).

Table 5.1

COEFFICIENT OF VARIATION FOR VACANCY DURATIONS IF EXPECTED DURATION WERE UNIFORM WITHIN SUBMARKETS DEFINED BY REGION AND TENURE: UNITED STATES, 1975

Submarket	Distribution of Annual Turnover[a] W_k	Average Vacancy Duration (weeks) $E_k(x)$	Moments of the Distribution of Vacancy Durations	
			First Moment $W_k\ E_k(x)$	Second Moment $W_k[E_k(x)]^2$
Northeast				
Owner	.032	8.7	.28	2.42
Renter	.137	5.8	.79	4.61
North Central				
Owner	.069	5.5	.38	2.09
Renter	.175	5.7	1.00	5.69
South				
Owner	.092	7.1	.65	4.64
Renter	.253	7.0	1.77	12.39
West				
Owner	.059	6.0	.35	2.12
Renter	.183	5.2	.95	4.94
Total	1.000	--	6.17	38.90

Coefficient of variation $= \sqrt{2.138(38.90)/(6.17)^2 - 1} = 1.088$

SOURCE: Equation (10); Table 2.1; and unit counts from *Annual Housing Survey: 1975, Part A*, Bureau of the Census, Series H-150-75A.

[a]Distribution of the product of the number of units and the turnover rate.

How much variation in expected duration across submarkets is necessary to explain the observed variation of realized vacancy durations? Table 5.3 provides one answer by showing hypothetical submarkets with expected durations ranging from 2 to 40 weeks. The coefficient of variation for vacancy durations is now 2.1, still not 2.3 but close enough to show the dramatic differences among submarkets that the 2.3 figure implies.

Table 5.2

COEFFICIENT OF VARIATION FOR VACANCY DURATIONS IF EXPECTED DURATION WERE UNIFORM WITHIN SUBMARKETS DEFINED BY LOCATION AND TYPE OF PROPERTY: ST. JOSEPH COUNTY, 1974, AND BROWN COUNTY, 1973

Submarket	Distribution of Annual Turnover[a] W_k	Average Vacancy Duration (weeks) X_k	Moments of the Distribution of Vacancy Durations: First Moment $W_k X_k$	Moments of the Distribution of Vacancy Durations: Second Moment $W_k X_k^2$
Central South Bend				
Owner	.053	21.2	1.12	23.82
Renter, single-unit	.063	9.8	.62	6.05
Renter, multiunit	.137	10.5	1.44	15.10
Rest of St. Joseph County				
Owner	.162	9.6	1.56	14.92
Renter, single-unit	.047	8.2	.39	3.16
Renter, multiunit	.125	8.0	1.00	8.00
Brown County				
Owner	.087	5.2	.45	2.35
Renter, single-unit	.044	4.5	.20	.89
Renter, multiunit	.282	3.9	1.10	4.29
Total	1.000	--	7.88	78.58

Coefficient of variation $= \sqrt{2.138(78.58)/(7.88)^2 - 1} = 1.306$

SOURCE: Equation (10); Table 3.1; and unit counts from the HASE baseline surveys of tenants and homeowners.

[a]Distribution of the product of the number of units and the turnover rate.

Many distributions of expected vacancy duration will give the same coefficient of variation as the distribution in Table 5.3. However, all will show most submarkets clustered at low expected durations and a few submarkets extending a long tail into very high expected durations. That pattern follows from the need to obtain a high

Table 5.3

COEFFICIENT OF VARIATION FOR VACANCY DURATIONS
IF EXPECTED DURATION WERE UNIFORM WITHIN
HYPOTHETICAL SUBMARKETS WITH GREATLY
DIFFERING MARKET CONDITIONS

Hypothetical Submarket	Distribution of Annual Turnover	Average Vacancy Duration (weeks)	Moments of the Distribution of Vacancy Durations	
			First Moment	Second Moment
A	.20	2	.40	.80
B	.25	3	.75	2.25
C	.30	4	1.20	4.80
D	.12	10	1.20	12.00
E	.06	20	1.20	24.00
F	.04	30	1.20	36.00
G	.03	40	1.20	48.00
Total	1.00	--	7.15	127.85

Coefficient of variation = $\sqrt{2.138(127.85)/(7.15)^2 - 1} = 2.085$

SOURCE: Equation (10).

coefficient of variation with Eq. (10) while keeping all expected durations nonnegative.*

The existence of a few submarkets with very high expected vacancy durations also makes sense economically. The shifts in demand that make some submarkets tight and others loose will not change expected durations symmetrically. No matter how tight the submarket, expected vacancy duration cannot fall below zero, and because of new construction or conversions it will rarely reach the theoretical minimum.

---

*To see that a distribution of nonnegative values must have a tail to the right for the coefficient of variation to exceed 1.0, examine the extreme case of a distribution with p fraction at zero and $1 - p$ fraction at unity. The mean value is $(0)(p) + (1 - p)(1) = 1 - p$, and the variance is $(p)[0 - (1 - p)]^2 + (1 - p)[1 - (1 - p)]^2 = p(1 - p)$, so the coefficient of variation is $\sqrt{p/(1 - p)}$. It can only exceed 1.0 if p exceeds 0.5, i.e., if the distribution has a tail to the right.

However, there is no limit on how large expected vacancy duration can be in loose submarkets. Excess supply could well stay on the market for some time before owners admitted defeat and accepted the losses involved in scrapping residential improvements.

VI. CONCLUSIONS

The annual average vacancy rate is the product of the annual average turnover rate and the annual average vacancy duration. Empirical evidence shows that average vacancy duration is the component that measures housing market condition. Apparently the turnover rate is a characteristic of demand only, telling nothing about the relationship between supply and demand.

If the turnover rate is stable, as in longitudinal analyses of a given housing market, the vacancy rate and vacancy duration are equally satisfactory measures of housing market condition. However, where the turnover rate varies, as it does in cross-sectional analyses, average vacancy duration is a better measure.

In 1975, the national average vacancy duration was 6.3 weeks for owner units and 6.1 weeks for renter units, while the vacancy rate was 1.2 percent for owner units and 6.0 percent for renter units. Using the average vacancy duration measure, we conclude that the owner and renter markets had essentially the same conditions in 1975. If we had used the vacancy rate measure, we would have concluded that the owner market was far tighter than the renter market. The two measures of market condition give different answers because the annual turnover rate varies from 9.9 per 100 owner units to 51.2 per 100 renter units.

The annual turnover rate was lowest in the Northeast region and highest in the West, for both owner and renter units. The vacancy rates in the Northeast were only two-thirds those in the West, because of the lower turnover rates, not tighter housing markets. The average vacancy duration shows that the market for owner units was considerably looser in the Northeast than in the West (8.7 vs. 6.0 weeks), and the market for renter units was slightly looser (5.8 vs. 5.2 weeks).

Applying the vacancy duration measure to the HASE sites, we found that market condition varies greatly by location (central South Bend, the rest of St. Joseph County, and Brown County) but only slightly by type of housing (owner, single-unit rental, and multiunit rental).

If we had used vacancy rate to measure market conditions we would have reached roughly the same conclusion about the variation of market condition by location, but we would have found even greater variation by type of housing.

St. Joseph County has a loose housing market, with an average vacancy duration of 12.7 weeks in central South Bend and 8.8 weeks in the rest of the county. Brown County has a tight housing market, with an average vacancy duration of only 4.2 weeks. Those durations bracket both the national average of 6.2 weeks and the North Central regional average of 5.6 weeks. The experimental sites are thus good places to test the effects of market conditions on the housing allowance program.

The annual average *complete* vacancy duration is the time from start to finish of all vacancies during a year. The average *interrupted* vacancy duration is the time from the start of a vacancy until it is interrupted by the survey question, "How long has this vacancy existed?" "Interruption bias" alone causes the average interrupted duration to equal one-half the average complete duration. That bias is countered, however, by a "duration bias," the greater likelihood that longer vacancies will exist on the survey date. In the United States in 1975, duration bias won handsomely, with the result that the average interrupted vacancy duration was three times the average complete vacancy duration.

A simple formula shows the combined effect of interruption bias and duration bias. The ratio of average interrupted vacancy duration to average complete vacancy duration equals $(1 + C^2)/2$, where C is the coefficient of variation for vacancy durations. If C is zero, there is pure interruption bias and the ratio equals one-half. Because there is no upper bound on C, there is no limit on how great the duration bias can be. Since the observed ratio is three, use of the formula shows that the coefficient of variation for vacancy durations in the United States in 1975 was 2.3.

The standard deviation of vacancy durations is thus 2.3 times as large as the average vacancy duration. The amount is particularly striking considering that, since vacancy duration cannot be negative, there can be only one long tail on the distribution of vacancy durations.

The large variation in vacancy durations is strong evidence that housing submarkets exist. If all units in the housing market had the same expected vacancy duration, then the distribution of vacancy durations would be exponential and the coefficient of variation would be 1.0. Only if submarkets exist can expected vacancy durations vary among housing units, and only then can the coefficient of variation for realized vacancy durations exceed 1.0.

That the coefficient of variation for vacancy durations greatly exceeds 1.0 also shows that market condition varied greatly across U.S. submarkets in 1975. The evidence does not define the submarkets but confirms that they exist and are important.

Future research on identifying submarkets can use the average vacancy duration measure as one test for submarkets. Although different submarkets need not have different average vacancy durations, this note shows that different average vacancy durations always imply the existence of submarkets. Furthermore, the method devised here for estimating the coefficient of variation for vacancy durations can be used to test for the existence of more detailed submarkets within submarkets already identified. Again, the converse is not necessarily true, but if the coefficient of variation for vacancy durations exceeds 1.0 for a housing submarket, then more detailed submarkets exist to be found.

Appendix A

ESTIMATING AVERAGE COMPLETE VACANCY DURATION

The text proved that the average duration of a housing vacancy equals the ratio of vacancy rate to turnover rate, provided that annual averages are used and that the annual pattern of turnovers and vacancy durations is stable. This appendix describes the methods used to estimate the vacancy and turnover rates, discusses the seasonal variation that makes annual averages necessary, and derives standard errors of estimate.

VACANCY RATE

This report uses the Bureau of the Census definition of vacancy rate: the number of unoccupied units that are for sale or rent divided by the number of units in the housing stock. Only year-round units are included in that ratio (because the occupancy status of seasonal housing is difficult to define, let alone measure), and the numerator does not contain units that are sold or rented but not yet occupied, units held for occasional use, or vacant units not on the market.

The usual way to estimate a vacancy rate is to survey housing units and find the proportion that are vacant. That is how the Census Bureau produced the vacancy rates in Table 2.1, and how we produced the vacancy rates for owner units in Table 3.1. For the rental units in Table 3.1, however, we were able to use HASE revenue accounts to estimate the vacancy rates by the percent of rent lost because of vacancy. Since the vacancy rate, by definition, only counts vacant units that are not yet rented, the second method is theoretically equivalent to the first.

We use the rent-loss method to estimate vacancy rates where possible because the standard error of the rent-loss method is only about 40 percent that of the proportion-vacant method. In other words, the proportion-vacant method requires a sample six times larger to achieve the same accuracy as the rent-loss method.

In using rent loss to estimate the vacancy rate, we could not simply use the average percent of rent lost because of vacancy. That would give a biased estimate whenever low-rent units had vacancy rates systematically different from high-rent units. To avoid that bias, we used weights proportional to the number of units on a property (instead of proportional to the property's rent), when averaging property-specific rent-loss rates over all properties in the analysis sample. We would like to make the adjustment at the unit level, but with landlord survey data it can only be done at the property level.

TURNOVER RATE

The turnover rate is the frequency with which vacancies occur. If we date vacancies by starting date, then annual turnover equals the number of times during a year that occupants move out of existing units plus the number of units newly entering the rental or sales market. Alternatively, if we date vacancies by ending date, then annual turnover equals the number of times during a year that occupants move into a unit plus the number of units removed from the rental or sales market. The two operational definitions are equivalent if the turnover process is annually cyclical.

We used the second operational definition, the frequency with which vacancies end, in estimating the turnover rates in this analysis. For the national and regional averages in Table 2.1 we summed the move-in rate (annual number of move-ins per housing unit) and the removal rate (annual number of units removed from the housing market relative to the number in the market). For the St. Joseph County and Brown County averages in Table 3.1, however, we used only the move-in rate because the HASE baseline studies cover only housing that existed all year.

Mobility histories in the HASE tenant/homeowner surveys provided the annual counts of move-ins for the experimental sites. We simply totaled the move-ins reported by the histories for the baseline year.*

---

*Previous HASE analyses of turnover on rental properties estimated the turnover rate by annual counts of move-outs reported by

The Census Bureau's annual housing survey for 1975 reported the number of households that moved into their housing unit during the 12 months preceding October 1975. However, that count includes only the last moves made during the year. To obtain total move-ins, we added an estimate of the prior moves made by households during the same year. It was calculated by multiplying the number of last move-ins in the nation or region by .08 for owner units and by .39 for renter units; those factors were obtained from HASE data.

Table A.1 presents the three components of the estimated national and regional turnover rates: last move-ins, prior move-ins, and removals. Table A.2 shows the data used to estimate the ratio of prior move-ins to last move-ins.

SEASONAL VARIATION

The Bureau of the Census estimates annual vacancy rate for its *Housing Vacancies* report by averaging the results of four quarterly estimates. Those results show very little seasonal variation in vacancy rates (see Table A.3). Presumably neither the number of households nor the number of housing units varies seasonally. The lack of seasonal variation in vacancy rates means that, if necessary, one can use the results of a vacancy survey in only one season to estimate the annual average vacancy rate. That was necessary for estimating the vacancy rates of owner units in St. Joseph and Brown counties.*

In contrast to the vacancy rate, the turnover rate varies greatly by season. Using utility company records of address changes for Brown

---

landlords; see *Third Annual Report of the Housing Assistance Supply Experiment*, pp. 64-65. Theoretically, the move-ins reported by tenants should equal the move-outs reported by landlords, for properties that exist all year. In practice, however, the landlord counts tend to be lower than the tenant counts, though not uniformly so. We do not know the reason for the differences but judge that the tenant survey is more accurate because it is more detailed. The tenant survey obtains a mobility history, complete with move-in dates and unit characteristics, while the landlord survey only obtains the landlord's count of annual move-outs for all units on his or her property.

*We did not need to resort to nonseasonality in estimating the annual vacancy rates of rental units because we had data covering the entire year's rent loss due to vacancies.

Table A.1

ANNUAL TURNOVER OF HOUSING UNITS BY TENURE AND REGION: UNITED STATES, 1975

Region	Number per 100 Housing Units			
	Components of Annual Turnover			Annual Turnover
	Last Move-ins[a]	Prior Move-ins[b]	Removals from Inventory[c]	
Owner Units				
Northeast	5.2	0.4	0.4	6.0
North Central	8.1	0.7	0.7	9.5
South	9.1	0.7	1.2	11.5
West	11.4	0.9	0.8	13.1
United States	8.4	0.7	0.8	9.9
Renter Units				
Northeast	25.2	9.8	1.5	36.5
North Central	36.0	14.0	2.0	52.0
South	39.5	15.5	2.2	57.2
West	43.2	16.9	1.5	61.6
United States	35.7	13.9	1.9	51.5
All Units				
Northeast	13.5	4.3	0.9	18.7
North Central	17.0	4.9	1.1	23.0
South	19.6	5.8	1.6	27.0
West	24.1	7.2	1.1	32.4
United States	18.4	5.5	1.2	25.1

SOURCE: *Annual Housing Survey: 1975*, Part A, Bureau of the Census, Series H-150-75A, Tables 1 and 5 in Secs. A through E.

[a]Number of households that moved into their units during the 12 months preceding October 1975.

[b]Estimated by multiplying last move-ins by .08 for owners and .39 for renters (see Table A.2 for the derivation of these factors).

[c]One-half the units removed from the inventory between October 1973 and October 1975.

Table A.2

ANNUAL MOBILITY OF HOUSEHOLDS BY TENURE: BROWN COUNTY, 1973, AND ST. JOSEPH COUNTY, 1974

	Percentage Distribution of Households by Number of Move-ins per Year							Ratio of Total Move-ins to Mobile Households[a]
	0	1	2	3	4	5+	All	
Owner								
Brown County	92.4	7.1	0.5	-	-	-	100.0	1.07
St. Joseph County	90.4	8.9	0.4	0.3	-	-	100.0	1.10
Average	91.4	8.0	0.4	0.2	-	-	100.0	1.08
Renter								
Brown County	50.0	35.7	11.1	2.2	0.5	0.5	100.0	1.38
St. Joseph County	54.1	31.6	11.2	2.5	0.4	0.2	100.0	1.40
Average	52.1	33.7	11.1	2.3	0.5	0.3	100.0	1.39

SOURCE: HASE baseline surveys of tenants and homeowners (excluding occupants of mobile homes, rooming houses, farmhouses, and federally subsidized units).

NOTE: Sample sizes are 2,833 renters and 900 owners in Brown County, and 2,133 renters and 641 owners in St. Joseph County.

[a]Ratio of the total number of move-ins during a year to the number of households making one or more moves during the year; e.g., for Brown County owners, [(1)(7.1) + (2)(0.5)]/[7.1 + 0.5] = 1.07.

County, we found that turnover rates are twice as high in summer as they are in winter (see Table A.4).

If we are correct in assuming that Brown County's seasonal variation in turnover rates also occurs nationally, then the evidence that vacancy rate is seasonally constant implies seasonal variation in average vacancy duration. To accommodate the larger number of summer turnovers without increasing the vacancy rate, average vacancy duration must be about half as great in summer as in winter.

That turnover affects vacancy duration during a year runs counter to this report's conclusion that annual turnover is a demographic characteristic that is independent of the annual average vacancy duration caused by market condition.

No available evidence suggests, nor does it seem plausible, that market condition varies seasonally with vacancy duration. The theoretical and empirical conclusions in this report are not affected by

Table A.3

VACANCY RATE BY SEASON:
UNITED STATES, 1967-76

Season	Vacancy Rate (%)	
	Owner Units	Renter Units
First quarter	1.11	5.85
Second quarter	1.07	5.92
Third quarter	1.17	5.90
Fourth quarter	1.15	5.56

SOURCE: *Housing Vacancies*, fourth quarter 1976, Bureau of the Census, Series H-111-76-4, Table 1, p. 1.

Table A.4

PERCENTAGE DISTRIBUTION
OF ADDRESS CHANGES BY
MONTH: BROWN COUNTY,
WISCONSIN, 1970-73

Month	Percent of Address Changes
January	5.5
February	5.4
March	6.4
April	6.0
May	7.4
June	10.1
July	10.1
August	9.7
September	12.3
October	10.7
November	8.7
December	7.7
Total	100.0

SOURCE: Compiled from connect-disconnect records of the utility company serving Brown County.

seasonal variation in turnover rates and vacancy durations. The proofs of theoretical relationships assume only that the annual cycles in turnover rates and vacancy durations are the same each year, and the empirical comparisons use annual averages.

STANDARD ERRORS OF ESTIMATE

To guard against spurious conclusions (ones that subsequent samples will fail to replicate), Tables 2.1 and 3.1 report standard errors of estimate. With sample sizes like ours of at least 100 observations, the estimated value of a parameter will differ from the true value by less than the standard error 68 times out of 100, and it will differ by less than 1.96 times the standard error 95 times out of 100.

When vacancy rate is measured by the proportion of units that are vacant on a given day, the standard error equals $\sqrt{v(1 - v)/n}$, where v = vacancy rate and n = sample size (in units). When vacancy rate is measured by the rent-loss rate, the standard error is $\sqrt{s^2/n}$, where s = standard deviation (root mean square deviation from the mean) of the rent-loss rate and n = sample size (in properties).

We estimate the standard error of the annual turnover rate by $\sqrt{t/n}$, where t = annual turnover per unit and n = sample size (in units). The formula assumes that the variance of annual turnover per unit equals the average, i.e., that turnovers have a Poisson distribution. Table A.5 demonstrates the validity of that assumption. Note, however, that the test is not perfect since it is done on move-ins per household instead of on turnovers per housing unit: the denominator is households instead of housing units, and the numerator does not include removals. Nevertheless, the test adequately defends the $\sqrt{t/n}$ formula for obtaining approximate standard errors of turnover rates.

Finally, we estimate the standard errors of average vacancy duration with the formula for error propagation under division: $s(v/t) = (v/t)\sqrt{[s(v)/v]^2 + [s(t)/t]^2}$, where $s(v/t)$ is the standard error of the ratio of vacancy rate, v, to turnover rate, t, and $s(v)$ and $s(t)$ are the standard errors of the vacancy and turnover rates.

The standard errors for the national and regional vacancy rates in Table 2.1 were computed by the Census Bureau and published along

Table A.5

ACTUAL VS. POISSON DISTRIBUTIONS OF HOUSEHOLDS BY NUMBER OF MOVE-INS PER YEAR: BROWN COUNTY, 1973, AND ST. JOSEPH COUNTY, 1974

Annual Move-ins per Household	Owner		Renter	
	Actual Distribution of Households	Poisson Distribution of Households	Actual Distribution of Households	Poisson Distribution of Households
0	91.4	91.2	52.1	51.5
1	8.0	8.4	33.7	34.1
2	.4	.4	11.1	11.3
3	.2	–	2.3	2.5
4	–	–	.5	.4
5+	–	–	.3	.2
All households	100.0	100.0	100.0	100.0
Average number of move-ins	.092	.092	.633	.633
Variance of the number of move-ins	.105	.092	.703	.633

SOURCE: Actual distribution from the average of Brown and St. Joseph counties in Table A.2. Poisson distribution from $e^{-m}m^{t}/t!$, where t = the number of move-ins per household per year, and m = average of t (known from the actual distribution).

with the vacancy rates. The accompanying notes on sample error explain that the vacancy rates come from the national Current Population Survey sample of 57,000 housing units visited monthly, and that the standard errors measure the effects of response and enumeration errors as well as sampling variability.*

The standard errors for the national and regional turnover rates in Table 2.1 are computed using the $\sqrt{t/n}$ formula, with sample sizes by region and tenure estimated as 1 unit out of every 1,366 in the population.** The national sample is slightly larger than that used for the Current Population Survey.

Housing Vacancies, annual statistics 1975, Bureau of the Census, Series H-111-75-5, pp. 8 and 10.

**The rule of thumb for sample sizes is given in *Annual Housing Survey: 1975*, Part A, Bureau of the Census, Series H-150-75A, pp. App-43, 44.

The standard errors for the vacancy and turnover rates in Brown and St. Joseph counties are estimated using the sample sizes given in Tables A.6 and A.7. The sample for owner vacancy rates is larger than that for owner turnover rates because all attempted interviews obtained occupancy status, whereas only completed interviews obtained turnover data. The sample of properties for computing renter vacancy rates by the rent-loss method is sometimes larger than the sample of units for renter turnover rates (because some landlords completed interviews when tenants did not), and sometimes smaller (either because tenants completed interviews when landlords did not or because multiunit properties have more than one tenant).

Table A.6

SIZE OF SAMPLES USED TO ESTIMATE VACANCY AND TURNOVER RATES FOR OWNER UNITS: ST. JOSEPH COUNTY, 1974, AND BROWN COUNTY, 1973

Location	Sample for Vacancy Rate Estimates[a] (units)	Sample for Turnover Rate Estimates[b] (units)
Central South Bend	328	164
Rest of St. Joseph County	814	477
Brown County	1,241	900
Total	2,383	1,541

SOURCE: HASE baseline surveys of homeowners.

[a]Owner units in the baseline sample, whether or not an interview was obtained (occupancy status was determined during the interview attempt, not in the interview).

[b]Owner units for which the occupant's mobility history was obtained in an interview.

Table A.7

SIZE OF SAMPLES USED TO ESTIMATE VACANCY AND TURNOVER RATES FOR RENTER UNITS: ST. JOSEPH COUNTY, 1974, AND BROWN COUNTY, 1973

Location and Property Type	Sample for Vacancy Rate Estimates[a] (properties)	Sample for Turnover Rate Estimates[b] (units)
Central South Bend		
Single-unit property	413	306
Multiunit property	408	463
Rest of St. Joseph County		
Single-unit property	448	355
Multiunit property	253	1,009
Brown County		
Single-unit property	642	725
Multiunit property	938	2,108
Total	3,102	4,966

SOURCE: HASE baseline surveys of landlords and tenants.

[a]Rental properties for which complete rent information was obtained in a landlord survey.

[b]Renter units for which a complete mobility history was obtained in a tenant survey.

Appendix B

ESTIMATING AVERAGE INTERRUPTED VACANCY DURATION

Interrupted vacancy duration is the time from the start of a vacancy to its "interruption" by a vacancy survey. The Census Bureau obtains the frequency distribution of interrupted vacancy durations using five closed intervals and one open interval (see Table B.1). To estimate the overall average interrupted vacancy duration, we assumed that the durations *within each interval* are distributed exponentially, so that the average duration in an interval is equal to:

$$E(i|a < i < b) = \frac{\int_{i=a}^{b} i\lambda e^{-\lambda i}}{\int_{i=a}^{b} \lambda e^{-\lambda i}},$$

where $E(i|a < i < b)$ = expected value of interrupted durations in an interval,

i = interrupted vacancy duration,

a = starting month of the interval,

b = ending month of the interval, and

λ = probability that a vacancy will end during a month.

We estimate the probability that a vacancy will end during a month by the inverse of the average vacancy duration. The national average vacancy duration is 6.2 weeks, or 1.43 months. Its inverse, to be used as λ in the equation above, is 0.7.

The resulting estimated averages by interval are given in the last two columns of Table B.1. Using them, we computed the overall average for the entire range of interrupted vacancy durations.

Table B.1

AVERAGE INTERRUPTED VACANCY DURATION:
UNITED STATES, 1975

Interrupted Vacancy Duration	Percentage Distribution of Vacant Units	Average Interrupted Vacancy Duration	
		Months	Weeks
Owner Units			
Less than 1 month	13	0.44	1.9
1 to 2 months	11	1.44	6.2
2 to 4 months	17	2.77	12.0
4 to 6 months	13	4.77	20.7
6 to 12 months	19	7.34	31.8
12 months or more	27	13.43	58.2
Entire range	100	6.33	27.4
Renter Units			
Less than 1 month	34	0.44	1.9
1 to 2 months	16	1.44	6.2
2 to 4 months	17	2.77	12.0
4 to 6 months	10	4.77	20.7
6 to 12 months	11	7.34	31.8
12 months or more	12	13.43	58.2
Entire range	100	3.75	16.3

SOURCE: *Housing Vacancies,* first through fourth quarters, 1975, Bureau of the Census, Series H-111-75-1 through 4.

NOTE: The distribution of vacant units is the average of those reported for each of the four quarters. The overall average duration is the sum of the detailed range averages weighted by the distribution of vacant units. See accompanying text for the method of estimating the detailed range averages.

Because the text showed that the average interrupted vacancy duration is considerably larger than the average complete vacancy duration, we know that the assumption of a negative exponential distribution is not correct. However, because we use the assumption only within each interval, the errors caused by the assumption are acceptable. In other words, if we knew the correct interval averages to use in Table B.1, they would not differ much from those obtained by using the exponential approximation.

Table B.2 shows the results of applying the method in Table B.1 to quarterly distributions of interrupted vacancy duration. The conclusion is that average interrupted vacancy duration does not vary seasonally. That contrasts with the conclusion in Appendix A that the average complete vacancy duration does vary seasonally.

Table B.2

AVERAGE INTERRUPTED VACANCY DURATION
BY SEASON: UNITED STATES, 1975

Season	Average Interrupted Vacancy Duration (weeks)	
	Owner Units	Renter Units
First quarter	26.8	16.0
Second quarter	28.3	16.4
Third quarter	26.9	16.6
Fourth quarter	28.8	16.1
Entire year	27.4	16.3

SOURCE: *Housing Vacancies*, first through fourth quarters, 1975, Bureau of the Census, Series H-111-75-1 through 4.

NOTE: Season averages were estimated from Census data using the method in Table B.1.

Table B.3 gives the Census Bureau estimates of the standard errors for the percentages in Table B.1. Because those percentages are uncertain, the vacancy durations in Table B.1 are also uncertain. To calculate the standard error of the average interrupted vacancy duration, we used the formula $\sqrt{\Sigma(i_k s_k/100)^2}$, where i_k = average interrupted vacancy duration for interval k and s_k = standard error of percent of vacant units in interval k. See the middle column of Table 4.1 for the resultant standard errors.

Table B.3

STANDARD ERRORS OF ESTIMATED PERCENTAGES OF VACANT UNITS BY INTERRUPTED VACANCY DURATION

Interrupted Vacancy Duration	Standard Error of Estimated Percent of Vacant Units	
	Owner	Renter
Less than 1 month	.8	.7
1 to 2 months	.7	.6
2 to 4 months	.8	.6
4 to 6 months	.8	.5
6 to 12 months	.9	.5
12 months or more	1.0	.5

SOURCE: *Housing Vacancies*, annual statistics 1975, Bureau of the Census, Series H-111-75-5, Table C, p. 11.

For the ratio of interrupted to complete durations given in the last column of Table 4.1, $R = E(i)/E(x)$, the standard error was computed using the rule for error propagation under division: the square of the relative error equals the sum of the squared relative errors of the numerator and denominator.

The standard error of the coefficient of variation, C, given in Table 4.2 equals the standard error of the ratio in Table 4.1 divided by the coefficient of variation. To see why that is so, we first invert Eq. (5) to yield $C = \sqrt{2R - 1}$. As R has a standard error, r, the formula becomes $C = \sqrt{2(R \pm r) - 1}$, which equals $\sqrt{(2R - 1) \pm 2r}$. Finally, using the rule that taking the square root halves the relative error, we conclude that $C = \sqrt{2R - 1} \pm r/\sqrt{2R - 1}$.

As reported in Table 4.2, the resulting standard error of estimate is only .05, which is very small compared with the 2.3 coefficient of variation of vacancy durations. However, the .05 figure includes only error due to sampling variability, not to measurement error.

At least three sources of measurement error affect our estimate of the coefficient of variation: (1) error in the estimate of prior moves used in calculating turnover rate (see Table A.2), (2) error in the interval averages for the distribution of interrupted vacancy duration (see Table B.1), and (3) error in the interrupted vacancy durations reported to the Bureau of the Census. The third error occurs because a vacant unit, by definition, has no occupant to interview. The Bureau must ask an "informed respondent" how long the unit has been vacant, and the answer may not have the accuracy that could have been obtained from an occupant. If such errors could be quantified, the revised standard error would surely be considerably larger than .05. Nevertheless, we judge it very unlikely that measurement errors are large enough to threaten the conclusion that the coefficient of variation for vacancy durations is greater than 1.0.

Appendix C

RELATION BETWEEN THE VACANCY-ENDING RATE AND THE VARIABILITY OF VACANCY DURATIONS

The vacancy-ending rate is the fraction of vacancies that end per unit of time, as dwellings are rented, sold, or removed from the housing market.* If the vacancy-ending rate of the surviving vacancies in a cohort has a consistent trend, a useful statement can be made about the variation of vacancy durations.

THEOREM: VARIATION OF VACANCY DURATIONS

If the vacancy-ending rate decreases (is constant, increases) over time from the start of the vacancy, then the coefficient of variation for vacancy durations is greater than (equal to, less than) 1.0.

REMARKS

The theorem is true regardless of the reason for the increase or decrease in vacancy-ending rate. Increases can be caused only by a change in the vacancy-ending rate for specific units in a cohort of vacancies. Decreases, however, can be caused either by a change in the vacancy-ending rate for specific units or by a sorting of non-uniform constant rates as the cohort of vacancies ages. If all vacancies do not have identical rates, vacancies with low ending rates will become an increasing proportion of the surviving vacancies, so the vacancy-ending rate for the cohort will decrease.

The text observes that the coefficient of variation for vacancy durations is greater than 1.0, argues that vacancy-ending rates might increase but cannot decrease for specific units,** and therefore

---

*This appendix is a limited adaptation of the discussion of failure rates for materials, structures, and devices in Richard E. Barlow and Frank Proschan, *Mathematical Theory of Reliability*, John Wiley & Sons, New York, 1965, pp. 22-33.

**Although landlords or owners, discouraged by a unit's long vacancy, might lower its rent or purchase price and thereby *increase* its vacancy-ending rate, they would not act to *decrease* the rate.

concludes that the sorting of nonuniform vacancy-ending rates must be causing the observed variation in vacancy durations. Finally, the text argues that the nonuniformity of vacancy-ending rates implies that submarkets exist. The theorem proved in this appendix is thus used twice in the text's argument--first to show that vacancy-ending rates that increase for specific units *could not* cause the observed variation in vacancy durations, and second to show that nonuniform constant vacancy-ending rates *could* cause the observed variation in vacancy durations.

The proof of the theorem uses exponentially distributed vacancy durations as a benchmark case; the case has a constant vacancy-ending rate and a coefficient of variation equal to 1.0. Then the proof uses three lemmas to show that cases on either side of the benchmark establish the theorem.

NOTATION

x = duration of a vacancy,

$f(x)$ = the density function, giving the fraction of vacancies that have duration x,

$S(x) = \int_{t=x}^{\infty} f(t)dt$ = the survivor function, giving the fraction of vacancies that last as long as or longer than x,

$r(x) = f(x)/S(x)$ = the vacancy-ending rate, giving the rate at which surviving vacancies end, as a function of survival time x,

$E(x^r) = \int_{x=0}^{\infty} x^r f(x)dx$ = rth moment of the distribution of vacancy duration; for example, $E(x)$ is the mean and $E(x^2) - [E(x)]^2$ is the variance about the mean, and

$C = \sqrt{E(x^2) - [E(x)]^2}/E(x)$ = the coefficient of variation for vacancy durations, the ratio of the standard deviation to the mean.

THE EXPONENTIAL BENCHMARK

If the density function is exponential, $f(x) = \lambda\, exp(-\lambda x)$, then the survivor function is also exponential, $S(x) = exp(-\lambda x)$, the mean is $E(x) = 1/\lambda$, the second moment is $E(x^2) = 2/\lambda^2$, the coefficient of variation is 1.0, and the vacancy-ending rate is a constant, $r(t) = \lambda$. That proves the constant vacancy-ending rate part of the theorem and, it turns out, establishes a benchmark with which to prove the rest.

LEMMA 1: SHAPE OF THE SURVIVOR FUNCTION

If and only if the vacancy-ending rate decreases (is constant, increases), then the survivor function is log convex (log linear, log concave).

Proof.

$$r(x) = \frac{f(x)}{S(x)} = \frac{-\frac{d}{dx} S(x)}{S(x)} = -\frac{d}{dx} \log S(x)$$

$$\frac{d}{dx} r(x) \underset{>}{\overset{<}{=}} 0 \quad \text{iff} \quad \frac{d^2}{dx^2} \log S(x) \underset{<}{\overset{>}{=}} 0$$

LEMMA 2: DEFINITION OF MOMENTS USING THE SURVIVOR FUNCTION

$$E(x^r) = r \int_{x=0}^{\infty} x^{r-1} S(x)dx$$

Proof. Integrate by parts.

$$\int_{t=0}^{\infty} g(dh) = gh\Big]_0^{\infty} - \int_{t=0}^{\infty} (dg)h ,$$

where

$$g = x^r \qquad dg = rx^{r-1}$$

$$h = 1 - S(x) \qquad dh = f(x)$$

$$E(x^r) = \int_{x=0}^{\infty} x^r f(x)dx = x^r[1 - S(x)]\Big]_0^{\infty} - \int_{x=0}^{\infty} rx^{r-1}[1 - S(x)]dx$$

$$= -x^r S(x)\Big]_0^{\infty} + r \int_{x=0}^{\infty} x^{r-1} S(x)dx ,$$

which establishes the lemma because $S(x) = 0$ beyond some finite x (no vacancy lasts forever).

LEMMA 3: COMPARISON OF THE SURVIVOR FUNCTION WITH THE EXPONENTIAL FUNCTION HAVING THE SAME MEAN

If the vacancy-ending rate decreases (increases), then the survivor function with mean $1/\lambda$ crosses $exp(-\lambda x)$ once from below (above).

Proof. $S(x)$ and $exp(-\lambda x)$ both start at 1.0 when $x = 0$. The curves cannot cross more than once, and if they do cross must do so from the stated direction because of the shape known from lemma 1. The curves must cross at least once because we specify equal means, and lemma 2 shows that the areas under the two curves in the positive quadrant are equal.

PROOF OF THE THEOREM

We have already established the constant vacancy-ending rate part of the theorem. To prove the rest we use lemmas 2 and 3.

If the vacancy-ending rate decreases (increases), then where $S(x)$ has mean $1/\lambda$ and x^* is the unique point (see lemma 3) at which $S(x)$ crosses $exp(-\lambda x)$, the result is

$$2 \int_{x=0}^{\infty} [x - x^*][S(x) - exp(-\lambda x)]dx \gtrless 0 ,$$

because when $x - x^*$ is negative then $S(x) - exp(-\lambda x)$ is negative (positive), and when $x - x^*$ is positive then $S(x) - exp(-\lambda x)$ is positive (negative)--making the integrand always positive (negative). The implication is that

$$2 \int_{x=0}^{\infty} x[S(x) - exp(-\lambda x)]dx \gtrless 0 ,$$

because lemma 2 shows that the equality of the means causes the term with x^* to be zero.

Finally, again using lemma 2, and recognizing that for the exponential distribution $E(x^2) = 2[E(x)]^2$, we find that

$$E(x^2) = 2 \int_{x=0}^{\infty} x\ S(x)dt \gtrless 2 \int_{x=0}^{\infty} x\ exp(-\lambda x)dx = 2[E(x)]^2 ,$$

which proves the theorem's implication that

$$C = \sqrt{\frac{E(x^2)}{[E(x)]^2} - 1} \gtrless 1 .$$