Inclusionary Zoning and the Development of Urban Land

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Abstract. Inclusionary zoning is a set of controls and incentives designed to encourage the production of affordable housing. In this paper, I present a dynamic model of developer behavior, and examine how a developer would respond to the variety incentives provided by inclusionary zoning. By aggregating the profit-maximizing actions of developers, I am able to predict how inclusionary zoning could affect market-level variables such as the housing supply and average rent. I am also able to predict how inclusionary zoning could affect urban aggregates such as the urban-rural boundary and density. This work complements recent empirical studies, which provide intriguing evidence concerning the effects from inclusionary zoning.

1 Introduction

Inclusionary zoning (IZ) is a set of controls and incentives designed to encourage the production of affordable housing. The common characteristic of all IZ programs is the requirement that builders allocate a specific proportion of their development activity to “affordable” housing. For mandatory programs, it is common that builders have the alternative of paying a one-time fee rather than participating. Many programs are voluntary or allow significant exemptions. Most IZ programs offer developer incentives to compensate for the anticipated reduction in revenue. One of the most common incentives, the density bonus, allows developers to build beyond the density ceiling. Other incentives to participate consist of impact fee waivers, fast-tracking of permits, and construction subsidies.

According to the Center for Housing Policy (2008), the first IZ programs were implemented in the Boston area in 1972, the San Francisco area in 1973, and the Washington DC area in 1974. It is difficult to identify the precise number of jurisdictions that have some type of IZ code today, but the estimate is approximately 300 local governments. Approximately half of the IZ programs have been adopted since the early

1990s. How successful has inclusionary zoning been in promoting the production of affordable housing? In the San Francisco area, the affordable units built under IZ represented 2.3 percent of new residential permits between 1980 and 2006 (Schuetz et al. 2008). In the Washington, D.C., area the share of production during the same period is 3 percent.

Inclusionary zoning is a response by planners to criticisms of the exclusionary effects of minimum lot size zoning policy. More recently, growth management policies have also come under attack for tightening urban housing markets. Inclusionary zoning is presented by many as a cost-effective means of encouraging the production of affordable housing and overcoming the potential market pressures of inflexible planning regulations. Indeed, it is by offering the permission to exceed the maximum ceiling imposed by the pre-existing code that most IZ programs compensate the builder. Schuetz et al (2007) find that jurisdictions are more likely to have IZ programs when they are larger and more affluent and have adopted other land-use regulations.

Opponents of inclusionary zoning argue that the additional costs imposed on developers will raise the price of market-rate housing and reduce the housing supply (Ellickson, 1981 and Emrath, 2008). Depending upon the IZ program, there may be no subsidies offered to the builder. It is, in effect, an unfunded mandate on developers to provide affordable housing. It is viewed by some as a price control with a more appealing label. The standard criticism is that developers will move to other areas where development is more profitable. An argument by some urban planners is that this incentive could lead to urban sprawl (Pendall, 2008).

Past Research. Despite the increasing popularity of inclusionary zoning in rapidly growing communities, there are many unanswered questions concerning its effectiveness. Three recent empirical studies provide intriguing evidence concerning the affordable units produced and spillover effects from inclusionary zoning. Schuetz et al. (2007) examine three areas: the Boston suburbs, San Francisco, and Washington D.C. They find that the effect of IZ varies by area and focus on two findings. First, in the San Francisco area, there is no evidence of any effects of IZ on either the price or production of single family homes. In the Boston suburbs, however, they find that IZ has led to a slight increase in the price and a decrease in the production of single-family homes. The authors attribute this difference to differences in the IZ policies themselves. Knapp et al. (2008) examine IZ in California and find that the measurable effects are: an increase in the supply of multifamily housing and an increase in the price of single-family homes. Means and Stringham (2008) found that California cities adopting IZ ordinances experienced a 20 percent increase in prices and a 10 percent in the housing stock between 1990 and 2000.

Contribution of this Paper. Other researchers have expressed the basic insight that inclusionary zoning is an intervention in the housing market and may have distortionary effects. I refine this insight by analyzing the impact of individual IZ parameters on residential investment. There is something to be gained by an atomistic treatment of developer behavior. First, I can more accurately compare the effect of a diverse set of instruments. Second, an analysis that formally breaks inclusionary zoning into its component parts allows planners to understand the interaction of inclusionary zoning with other land use policies. Third, a formal model of the developer's objective function provides insight into how developers would react to voluntary provisions and exemptions, which is an important aspect of IZ programs. Fourth, by aggregating the profit-maximizing actions of developers, I predict how IZ affects market-level variables such as the housing supply and average rent. Fifth, I predict how IZ would affect urban aggregates such as the urban-rural boundary and density. A textbook model of supply and demand does not provide the same level of insight into urban form. Finally, this theory complements empirical research. It is admittedly difficult to interpret empirical results of inclusionary zoning because of the complexity of the policy. It is hoped that this theoretical work will assist in the task of explaining empirical research.

The paper is organized as follows: Section 2.1 presents a dynamic model of developer behavior, in which the developer chooses the timing of development to maximize the value of urban land in a growing economy. In Section 2.2, I explain how I model inclusionary zoning. In section 3, I analyze the effect of the primary elements of IZ, affordability requirements and bonus densities, on the timing of development when there is a binding ceiling on the density of development that restricts the density of development below the optimal level of development. In sections 4 and 5, I analyze additional characteristics of IZ programs. In section 6, I discuss the effect of affordability requirements and density bonuses on urban aggregates. In section 7, I discuss the case of flexible density. In section 8, I a present summary and possible extensions of this work.

2 Modeling Approach

In Section 2, I present a model of land development and explain how it can be adapted to analyze the effects of inclusionary zoning on residential investment.

2.1 Basic Model of Land Development

In this section, I outline a model of developer behavior in which the owner of a plot of land chooses the optimal time and density of development in order to maximize the value of land. Such a model was advanced by Arnott and Lewis (1978) and adaptations have been used by others to analyze the impacts of taxes, rent control, and zoning, and risk and uncertainty. This particular section concentrates on a description of the model absent of any regulation.

The Value of a Unit of Land

The value of one unit of developed residential land, $P^d(t)$, at time t is the net present value of rents, from time t to infinity times the density of housing, h, built on that plot of land. The rent on a unit of housing, $R(s)$, varies over time only.

$$P^d(t) = h \cdot \int_t^\infty R(s) e^{-r(s-t)} ds$$

Residential investment is infinitely durable and is also irreversible. There is no depreciation of the property or redevelopment. In other words, once built, housing will last forever.

The value of vacant land, $P^v(t)$, at time t is the net present value of rents at time from the time of development T less the cost of construction, $C(h)$, which depends on the density of housing. For the sake of simplicity, I assume no agricultural rents.[1]

$$P^v(t) = h \cdot \int_T^\infty R(s) e^{-r(s-t)} ds - C(h) \cdot e^{-r(T-t)}$$

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[1] One goal of a post-conference analysis is to add a return to agricultural (vacant) land to determine whether the results change.

Derivation of the Cost Function

The price of one plot of vacant land is derived from one of a developer with L units of land.[2] Housing, H, is produced with land, L, and capital, K, as reflected by the production function, $H(K,L)$. At the time of development the landowner must pay for the capital, K, used in the production of housing at a price of p per unit. To simplify notation, I measure capital in units such that the price of capital is normalized to one. To simplify the optimization problem, I examine the problem of maximizing the value of one plot of land. To do so, divide both sides of the objective function by the number of units of land, L. Assuming that the production function is homogeneous of degree one, the production function for a plot of land would be $h(k)$, where k is the capital-to-land ratio and $h(k)$ is the resulting output. I will express the optimization problems in terms of housing density, h, where the cost of density, $C(h)$, is simply the inverse of the production function, $C(h) = h^{-1}(h)$.

Optimization Problem

Real estate developers choose the timing of development and density of development to maximize the value of a parcel of vacant land:

$$\max_{T,h} P^v(t) = h \cdot \int_T^\infty R(s)e^{-r(s-t)}ds - C(h) \cdot e^{-r(T-t)}$$

The first-order condition with respect to the time of development, T, would be

$$P_T = -h \cdot R(T) + rC(h)$$

This first-order condition implies that the rent on a unit of housing at the optimal time of development, $R(T)$, must equal the average capital cost of development, $r\,C(h)/h$. The second-order condition with respect to timing is:

$$P_{TT} = -hR'(T) < 0$$

For development to be optimal in the future, rents must increase over time. Combined with the first-order condition for timing, the second-order condition implies that developers will wait to develop until the rent reaches the hurdle rent. Otherwise, if rents

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[2] This expression is $P^v(t) \cdot L = H(K,L) \cdot \int_T^\infty R(s)e^{-r(s-t)}ds - pK \cdot e^{-r(T-t)}$.

are not increasing, then development will either occur immediately or not at all. The first-order condition with respect to housing density is:

$$P_h = \int_T^{\infty} R(s) e^{-r(s-T)} ds - C'(h) = 0$$

Developers invest in density until the present value of rents from one unit of housing equals the marginal cost of building one more unit. The second-order condition with respect to density is

$$P_{hh} = -C''(h) < 0$$

For there to be a local maximum, the cost function is such that the marginal costs increase with output.

<u>Note to the discussant/reader: the following paragraph is not essential for understanding Sections 3 and 6, which will be the focus of my conference presentation.</u> A final condition for a local maximum is that the Jacobian must be positive:

$$J = P_{hh} P_{TT} - P_{hT} P_{Th} > 0$$

Suppose that $h(k) = k^{\gamma}$ and that the costs of capital are unitary, then I could express the cost of density as $C(h) = h^{1/\gamma}$ and $C'(h) = 1/\gamma \cdot h^{1/\gamma - 1}$. The Jacobian can be simplified to the below expression:

$$J = \frac{G(T)^2}{\gamma} \cdot \left(\frac{R'(T)}{r} \cdot \frac{1}{G(T)} - \gamma \right) > 0$$

where *G(T)* is defined as the present value of the rent growth at the time of development.[3] This expression will be positive for constant (linear) and decelerating rent growth. With accelerating growth, this is not assured because there may not be a local maximum, but instead a corner solution.[4] This is true when there is no additive portion of the rent.

[3] This substitution is allowed only as long as there is a solution for the optimal time of development (*T*) from the first-order condition with respect to *T*.

[4] For example, Capozza and Li note that with geometric growth and a Cobb-Douglas production function, there will be no local maximum: development occurs either immediately or never.

When there is an additive portion of the rent, such as a location premium,[5] then there will be a local maximum as long as $r > g/(1-\gamma)$, where g is the growth rate.[6]

The Concept of the Hurdle Rent

A concept used throughout the paper to understand optimal timing is that of the hurdle rent. The optimal time of development will be determined when the rent on a plot of land surpasses a critical level. In the discussion, above, I found that the critical level is equal to the interest rate times the average cost of development. The general point of models of the optimal timing of investment is that whenever the average benefit of a project increases over time relative to its costs, then the project begins sooner; and when costs increase relative to benefits then the project will be delayed.

In a model of flexible density this expression can be simplified by substituting the first-order condition with respect to density.[7] This version of the paper does not develop such an expression because the prevailing assumption will be that density is bounded. Complexity is introduced when there are multiple decision variables in the objective function

2.2 Modeling Inclusionary Zoning

Inclusionary zoning is a development regulation that includes many parameters. Inclusionary zoning requires developers to set a portion, λ, of their development to more affordable housing. A typical required share of affordable units is 10 percent, although there is one jurisdiction in the San Francisco area that requires a 25 percent share (Schuetz et al, 2008). The affordable units are set aside for low- or moderate-income households. Most regulations allow for a distribution of different income classes, ranging between 30 percent and 70 percent of the Area Median Income.[8] In this section, I assume that the rents on inclusionary housing are a fraction, equal to θ, of the market rent. There

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[5] In this case the rent function would be Such as $R(t,z) = R(0,0)\cdot e^{gt} - z$ where t is time and z is distance from the center of the city.

[6] With this particular set of assumptions, the center of the city would never be developed or would be developed immediately.

[7] See for example Arnott and Lewis, 1979 or Capozza and Li, 1994.

[8] HUD defines “Very Low Income” as less 50 percent of the area median family income and “Low Income” as from 50 to 80 percent of the area median family income (U.S. Department of Housing and Development, 2009).

is a lot of variation among programs, but generally, an IZ ordinance will stipulate that the rents are set such that they are not more than 30% of the tenant’s income, which is affordability standard used by HUD.[9]

The depth of the subsidy is modeled as a constant proportion, θ. The ratio of the allowed income share of IZ residents spent on housing to the market rent does not change. Note that the implicit assumption behind this simplification is that the residents of market-rate housing consume housing as a constant proportion of their household budget. In a world of property cycles, the ratio θ will change with short-run fluctuations of housing prices but over the long-run the housing market is driven by income growth. Thus, tents of inclusionary housing grow with the market because income drives housing prices.

The revenue on a unit of development at time *t*, would be: $(1-\lambda)h \cdot R(t) + \lambda h \cdot \theta R(t)$, where the first additive term is the revenue from market-rate housing and the second additive term is the revenue from affordable housing. I will simplify this by defining a new term, *α*, which represents the restrictiveness of inclusionary zoning. In this case, if $\alpha \equiv \lambda \cdot (1-\theta)$, which is between zero and one, then the revenue from a development at time *t* would be $(1-\alpha) \cdot h \cdot R(t)$. If the required set-aside increases or the targeted income level decreases, then this measure α will increase and diminish the revenues from a project.

An incentive for participating in inclusionary zoning is usually provided by local governments. In this paper, I examine the situation where a developer faces both maximum density zoning and inclusionary zoning regulations. In this case, developers are allowed to build above the maximum density at a bonus density percentage equal to *β*. For example, in the Washington D.C. area, the density bonuses range from 10 to 25 percent (Schuetz et al., 2008).

If the optimal density, as determined in the first-order condition with respect to density, is above the maximum allowed density, as represented by $\bar{h}$, then density will

[9] Many ordinances are vague concerning the target income group. For example, some may state only that the IZ units are reserved for “moderate income" or "low income" households without further precision. In such cases, it's difficult to identify the income base for calculating an affordable rent. (V. Been, New York University, New York, NY, personal communication to author, 2009.)

not be a decision variable in the absence of inclusionary zoning. This scenario, where $h > \bar{h}$, is the default for most of the analysis. One can only be certain, however, that density will be fixed under inclusionary zoning when the optimal density is greater than what the density bonus would allow, $h > (1+\beta)\cdot\bar{h}$.

A developer is not obliged to use the entire density bonus if the preferred intensity of development is less than what is permitted. The optimal density could be below the maximum density in the absence of the density bonus, in which case, density will be a decision variable. If the optimal level of development is between the maximum density and the maximum density plus the bonus, $\bar{h} < h < (1+\beta)\cdot\bar{h}$, the density will be a choice variable. In this case, we know that there will be an increase in density but do not know whether the entire bonus will used.

3 Inclusionary Zoning with Binding Density Ceiling

In this section, I analyze the impact of inclusionary zoning on the timing and density of residential development when the density ceiling is binding. This scenario is one of the most prevalent because the need for inclusionary housing is likely to be felt where the exclusionary zoning practices restrict the intensity of development. The only decision variable is the timing of development.

There are countervailing effects of inclusionary zoning on the profitability of development: a proportional reduction of revenue reduces the value of a development project but building at a higher density will increase the value of the project. The density bonus is a proportion equal to β. The reduction in revenue from a unit of housing density due to inclusionary zoning is a proportion equal to α. The density constraint is $\bar{h}$. In this case, I assume that the density constraint is binding, $h < (1+\beta)\cdot\bar{h}$. The price of developed land would be:

Equation 1

$$P^d(t) = (1+\beta)\cdot\bar{h}\cdot\int_t^\infty (1-\alpha)\cdot R(s)e^{-r(s-t)}ds$$

The price of vacant land would be:

Equation 2

$$\max_T P^v(t) = (1+\beta)\cdot \bar{h} \cdot \int_T^{\infty} (1-\alpha)\cdot R(s)e^{-r(s-t)}ds - C\left((1+\beta)\cdot \bar{h}\right) e^{-r(T-t)}$$

The first-order condition with respect to timing is:

Equation 3

$$P_T = -(1+\beta)\cdot \bar{h} \cdot (1-\alpha)\cdot R(T) + rC\left((1+\beta)\cdot \bar{h}\right) = 0$$

Equation 3 can be arranged to show the optimal time of development as determined by the hurdle rent:

$$R(T) = r \cdot \frac{C\left((1+\beta)\cdot \bar{h}\right)}{(1+\beta)\cdot \bar{h}} \cdot \left(\frac{1}{1-\alpha}\right)$$

Development occurs when the average rent from a unit of housing equals the capital cost of the average cost of development times an inflation factor. The average cost of construction is determined by the housing density allowed with the addition of the density bonus. The average rent collected by a landlord declines with inclusionary zoning. The second expression on the right-hand side (in large parentheses) represents the need for a higher hurdle rent to cover this decline in revenue: the market rent at the time of development is larger in order to compensate for the proportional reduction of rent revenue from inclusionary zoning.

The second-order condition with respect to the optimal time of development is:

$$P_{TT} = -(1+\beta)\cdot \bar{h} \cdot (1-\alpha)\cdot R'(T) < 0$$

Rents need to grow for development to occur in a continuous fashion.

I am interested in the effects of the program variables, the share of affordable housing and the density bonus, on developer behavior. To arrive at comparative static results, I apply the implicit function theorem to Equation 3. The change with respect to the affordable revenue share is given by:

$$\frac{\partial T}{\partial \alpha} = -\frac{P_{T\alpha}}{P_{TT}} = -\frac{(1+\beta)\cdot \bar{h} \cdot R(T)}{-(1+\beta)\cdot \bar{h} \cdot (1-\alpha)\cdot R'(T)} = \frac{R(T)}{(1-\alpha)\cdot R'(T)} > 0$$

Reducing a developer’s revenues; either by requiring a greater share of the development to be affordable housing or by requiring that affordable rents be lower; will delay development when density is not a choice variable. It takes longer for the average rent to reach the required hurdle rent to pay for the average capital costs of development.

The change in timing with respect to the density bonus is given by applying the implicit function theorem to Equation 3.

$$\frac{\partial T}{\partial \beta} = -\frac{P_{T\beta}}{P_{TT}} = \frac{-(1-\alpha)R(T) + rC'\left((1+\beta)\cdot \bar{h}\right)}{(1+\beta)\cdot(1-\alpha)\cdot R'(T)}$$

The denominator is positive because rents are increasing but the sign of the numerator is not obvious. From the first-order condition with respect to *T*, Equation 3, it is apparent that I can substitute $rC\left((1+\beta)\cdot \bar{h}\right)/(1+\beta)\cdot \bar{h}$ for $(1-\alpha)R(T)$ in the above equation to get:

$$\frac{\partial T}{\partial \beta} = \frac{r}{(1+\beta)\cdot(1-\alpha)\cdot R'(T)} \cdot \left(C'\left((1+\beta)\cdot \bar{h}\right) - \frac{C\left((1+\beta)\cdot \bar{h}\right)}{(1+\beta)\cdot \bar{h}} \right) > 0$$

The result depends on the term in brackets since the first multiplicative term is positive. The first term in brackets is the marginal cost of building $(1+\beta)\cdot \bar{h}$ units on a plot of land less the average cost of building at a density of $(1+\beta)\cdot \bar{h}$. Since marginal costs are assumed to be increasing,[10] the term in brackets is positive: the cost of building one more unit is always greater than the average cost of the previous units. Thus, increasing the bonus delays development when builders are restricted by density ceilings. This may seem counterintuitive. However, increasing the bonus raises revenues linearly while it increases the costs of development exponentially. While the bonus is a benefit to the developer, it becomes optimal to wait until rents rise to match the relatively higher cost of developing one unit of density.

The effect on vacant land prices. When the maximum density ceiling is binding, the affordable housing provision of inclusionary zoning and the density bonus will delay development. Density will increase to the maximum allowed by the density bonus. Although both of the parameters examined have the same impact on a developer's behavior under a binding density ceiling, they have the opposite effect on the profitability of a project. Requiring construction of affordable housing and lower target

[10] See the second-order condition with respect to density in Section 2.1.

incomes for the affordable housing units reduces the profitability of development. Providing a density bonus when builders are restricted by existing regulations increases the value of land because it allows a density of development that is closer to the optimal density.

The impact on prices of reducing the revenue from one unit of density will be to reduce the price of land. It is straightforward to show this for the affordable housing requirement

$$\frac{\partial P^v(t)}{\partial \alpha} = -(1+\beta)\cdot \bar{h} \cdot \int_T^\infty R(s) e^{-r(s-t)} ds$$

The direction of the effect of the bonus depends on whether the net present value of rents from a unit of density is greater than the marginal cost of building to the density allowed by the bonus.

$$\frac{\partial P^v(t)}{\partial \beta} = \bar{h} \cdot e^{-r(T-t)} \cdot \left[\int_T^\infty (1-\alpha)\cdot R(s) e^{-r(s-T)} ds - C'\left((1+\beta)\cdot \bar{h}\right)\right]$$

From the first- and second-order conditions with respect to density (see Section 2.1), I know that the above expression is positive when density is below the optimum.[11] Increasing the density bonus will raise the price of land as long as the density ceiling is below the optimal density.

4 Mandatory or voluntary participation

This section analyzes the voluntary/mandatory participation aspect of inclusionary zoning programs. I will continue with the assumption that the density ceiling is binding.

4.1 Voluntary Participation

According to the Center for Housing Policy (2008), most IZ programs are mandatory in the areas studied by Schuetz et al. (2007): 93 percent in the San Francisco area, 58 percent in suburban Boston, and 80 percent in the Washington D.C. area. Many builders would argue, however, that participation should be voluntary to ensure that

[11] Note that in the above derivatives the timing of optimal development is also a function of the policy variables. This does not show up in the derivatives because the first-order condition with respect to timing is equal to zero.

inclusionary zoning expand rather than limit builder's opportunities. Housing advocates may be concerned that if inclusionary zoning is not mandatory, then there would be no participants and thus no affordable housing units built.

To understand the implications of voluntary participation, the question to be answered is whether the incentives offered by IZ counter-balance the reduction in revenue. Developers will participate as long as the value of the project under inclusionary zoning, P_{IZ}^{v}, is as great as one that is not subject to IZ, P_{0}^{v}. For developers to participate,

$$P_{IZ}^{v} \geq P_{0}^{v}$$

where *IZ* signifies the presence of inclusionary zoning and *0* signifies the absence of inclusionary zoning.

The analysis of land prices can be made easier by separating it into components. I use integration by parts to do this for the present value of the future stream of rents,

$$\int_{T}^{\infty} R(s) e^{-r(s-t)} ds = \left(\frac{R(T) + G(T)}{r} \right) \cdot e^{-r(T-t)}$$

where $G(T)$ is the present future value of the growth of rents at the time of development,

$$G(T) = \int_{T}^{\infty} R'(s) e^{-r(s-T)} ds$$

Substituting this expression into Equation 2 and then substituting the hurdle rent from Equation 3 yields the following expression for the price of vacant land when the developer participates in the inclusionary zoning program:

$$P_{IZ}^{v}(t) = (1-\alpha) \cdot (1+\beta) \cdot \bar{h} \cdot \frac{G(T^{IZ})}{r} \cdot e^{-r(T^{IZ}-t)}$$

The time of development under inclusionary zoning is indicated as T^{IZ}. Identical steps lead to a similar expression for the price of land when inclusionary zoning is absent (α=0, β=0):

$$P_{0}^{v} = \bar{h} \cdot \frac{G(T^{0})}{r} \cdot e^{-r(T^{0}-t)}$$

The time of development in the absence of inclusionary zoning is indicated as T^0. Thus, a developer will participate in inclusionary zoning when:

$$(1-\alpha)\cdot(1+\beta)\cdot G\left(T^{IZ}\right)\cdot e^{-r\left(T^{IZ}-T^0\right)} \geq G\left(T^0\right)$$

If the growth of rents is linear, then this condition reduces further to $(1-\alpha)\cdot(1+\beta)\cdot e^{-r\left(T^{IZ}-T^0\right)} \geq 1$. This condition can be used to calculate the density bonus that would offset the reduction in rents:

$$\beta \geq \frac{e^{r\left(T^{IZ}-T^0\right)}}{(1-\alpha)} - 1$$

I have shown in Section 3 that inclusionary zoning delays development ($T^{IZ}>T^0$) when the regulatory environment is characterized by an affordable housing share, a density bonus, and a binding density ceiling. Thus, the numerator of the fraction on the left-hand side is greater than one. There are two lessons to be learned from this section. The first is that, when the density ceiling is binding, the density bonus can offset the reduction in rent revenue from inclusionary zoning. The second lesson is that a naïve percentage calculation of what the density should be would underestimate the necessary density bonus. The density bonus has to more than offset the rent reduction because of the impact that inclusionary zoning has on the timing of development.

4.2 Minimum Project Size

One provision of many inclusionary zoning regulations is that smaller developments are excluded from inclusionary zoning. In this paper, I model this provision as a density, $\tilde{h}$, below which inclusionary zoning does not apply. The question I try to answer in this section is under which conditions developers will build at that lower density to escape inclusionary zoning. Developers will choose inclusionary zoning if

$$P_{IZ}^{v} \geq P_{\tilde{h}}^{v}$$

where *IZ* signifies the presence of inclusionary zoning and $\tilde{h}$ signifies the absence of inclusionary zoning at a lower intensity of construction.

By definition, inclusionary zoning will increase density. To determine the impact on the timing of development, consider the following expressions derived from Equation 3:

$$R\left(T^{IZ}\right) = \frac{1}{1-\alpha} \cdot \frac{rC\left(\left(1+\beta\right)\cdot \bar{h}\right)}{\left(1+\beta\right)\cdot \bar{h}} \text{ and } R\left(T^{\tilde{h}}\right) = \frac{rC\left(\tilde{h}\right)}{\tilde{h}}$$

From the properties of the cost function and because α is less than one, I know that $R\left(T^{IZ}\right) > R\left(T^{\tilde{h}}\right)$ and thus $T^{IZ} > T^{\tilde{h}}$. The inclusionary zoning project will be built later and closer to the optimal density than the minimum project size. It is not obvious, however, which is the better decision.

Following the analysis in Section 4.1, the above condition can be expressed as:

$$\left(1-\alpha\right)\cdot\left(1+\beta\right)\cdot \bar{h} \cdot G\left(T^{IZ}\right) \cdot e^{-r\cdot T^{IZ}} \geq \tilde{h} \cdot G\left(T^{\tilde{h}}\right) e^{-r\cdot T^{\tilde{h}}}$$

The present value of rent growth on a development build under inclusionary zoning must be greater than or equal to the present value of rent growth on a development at the lower density to escape inclusionary zoning. Similarly to the section above, I will assume that rent growth is linear to simplify the expression above and express it as condition for the ratio of development intensity:

$$\left(1-\alpha\right)\cdot\left(1+\beta\right)\cdot e^{-r\cdot\left(T^{IZ}-T^{\tilde{h}}\right)} \geq \tilde{h}/\bar{h}$$

For a developer to choose inclusionary zoning over the minimum project size, the left-hand side of the above expression must be greater than the ratio of the minimum project size to the maximum allowed density. The condition is more likely to be satisfied the lower the offset is, the higher the bonus, and the smaller the ratio of regulated densities. The density bonus does not need to more than compensate for the rent reduction as it does in the case of voluntary provision because the alternative to inclusionary zoning is building at a lower and less optimal density. Note however, that one also has to consider the impact of the policy on the timing of development. I know from the hurdle rents that the inclusionary zoned development will be built later, $T^{IZ} > T^{\tilde{h}}$. Thus, the third term on the left-hand side, $e^{-r\cdot\left(T^{IZ}-T^{\tilde{h}}\right)}$, is less than one. As shown in the section above, a naïve calculation would underestimate the density bonus and overestimate the rent offset that would encourage developers to participate in inclusionary zoning.

5 Incentives

I have only considered one of the most prevalent incentives given to participants of inclusionary zoning programs. Others which are common and analyzed in this section are the impact fee waiver and fast-tracking of development permits.

5.1 Impact Fee Waiver

One of the many incentives that are offered is a reduction of impact fees. In this analysis, I consider the impact of a fee on land,[12] charged at the time of development. In this case, the objective function under a binding density ceiling is:

$$\max_{T} P^{v}(t) = (1+\beta)\cdot\bar{h}\cdot\int_{T}^{\infty}(1-\alpha)\cdot R(s)e^{-r(s-t)}ds - \left[C\left((1+\beta)\cdot\bar{h}\right) + F\right]e^{-r(T-t)}$$

The first-order condition for the optimal time of development is:

$$P_{T} = -(1+\beta)\cdot\bar{h}\cdot(1-\alpha)\cdot R(T) + rC\left((1+\beta)\cdot\bar{h}\right) + rF = 0$$

The first-order condition can be arranged to show the optimal time of development as determined by the hurdle rent:

$$R(T) = r\cdot\frac{C\left((1+\beta)\cdot\bar{h}\right)+F}{(1+\beta)\cdot\bar{h}}\cdot\frac{1}{1-\alpha}$$

It is apparent from the expression of the hurdle rent that the imposition of an impact fee will delay development and that the delay will be heightened by the size of interest rate and the offset provision; and reduced by the permitted density of development. To confirm this, I apply the implicit function theorem to the first-order condition:

$$\frac{\partial T}{\partial F} = -\frac{P_{TF}}{P_{TT}} = \frac{r}{(1+\beta)\cdot\bar{h}\cdot(1-\alpha)\cdot R'(T)} > 0$$

Reducing the impact fee on land would hasten development under inclusionary zoning. Depending on the size of the fee, a reduction of the impact fee could be used to neutralize the effect of the affordable housing provisions on development timing. However,

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[12] For other variations, see McFarlane (1999).

whether doing this is good economic policy is questionable. The purpose of an impact fee is to pay for the marginal cost of the development on the infrastructure and public services of the city. Explicitly reducing impact fees, while it would hasten the construction of housing, would lead to the suboptimal provision of public goods.[13] Indeed, it is the absence of marginal-cost pricing that leads to exclusionary zoning.

5.2 Fast-tracking of development permits

Another incentive for participating in inclusionary zoning programs is the fast-tracking of development. There are two ways in which delays can be costly to a developer. First, delaying the time of development beyond the optimal time reduces the value of a project. Second, there may be significant holding costs of land to a developer between the permit application and the completion of construction.

In this model of land development, and without holding costs, a delay will be costly only when it is unexpected. An unexpected delay would lower the value of land by depriving the landowner of urban rents greater than the average capital cost during the delay. If, through experience, developers know that there will be a delay, then they can account for the postponement by applying for a permit earlier than they would otherwise. If there were a one-year delay, then the developer would apply for a permit one year ahead of when the permit is needed. This intuitive result but can be shown formally. For example, if the probability of a delay is p^D and the delay is of length D then the developer of a project with rents $R(s)$ and a lump-sum cost of construction, C, would face the following objective function:

$$\max_T P^v(t) = p^D \cdot \left[\int_{T+D}^{\infty} R(s)e^{-r(s-t)}ds - C \cdot e^{-r(T+D-t)}\right] + \left(1 - p^D\right) \cdot \left[\int_{T}^{\infty} R(s)e^{-r(s-t)}ds - C \cdot e\right.$$

The hurdle rent would then be:

$$R(T) = rC - \frac{p^D \cdot e^{-rD}}{1 - p^D \cdot \left(1 - e^{-rD}\right)} \cdot \int_{T}^{T+D} R'(s)ds$$

If there were a 100% chance of delay, the fraction on the right-hand side would be one. The integral on the right-hand side is only the growth in rents from time T to time $T + D$.

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[13] For an analysis of impact fees in a similar model that explicitly addresses the efficiency aspects of infrastructure finance, see Brueckner (1997).

In the case of linear growth, g, the growth over that period is g X D. The expression reduces to $R(0)+gT=rC-gD$. An increase in delay will speed up the application for the permit and have a neutral effect on the timing of development.

More is needed for fast-tracking of permits to be a useful incentive. One such condition would be uncertainty. In this model, developers are able to predict the future rents. However, with uncertainty an artificial delay is likely to distort the actual development decision. Developers will postpone until they are sure that rents can't drop below what is optimal while waiting for the permit.

Fast-tracking of permits will be a valuable incentive regardless of the degree of uncertainty when there are waiting costs, w, for every moment of delay. These could be holding costs or processing costs that are borne from permit application to development. The objective function is:

$$\max_T P^v(t)=(1+\beta)\cdot\bar{h}\cdot\int_T^{\infty}(1-\alpha)\cdot R(s)e^{-r(s-t)}ds-C\left((1+\beta)\cdot\bar{h}\right)\cdot e^{-r(T-t)}-\int_{T-D}^{T}we^{-r(s-t)}ds$$

I derive a hurdle rent:

$$R(T)=\frac{1}{1-\alpha}\cdot\frac{rC\left((1+\beta)\cdot\bar{h}\right)+w\cdot\left(e^{rD}-1\right)}{(1+\beta)\cdot\bar{h}}$$

This expression is similar to the hurdle rent in the present of the lump-sum fee (see section above). The hurdle rent and thus the time of development will increase with both the holding costs and the length of delay. Fast-tracking permits would act to counterbalance the dilatory effects of inclusionary zoning when the density ceiling is binding. Of course, if all developers were subject to inclusionary zoning, then it would be impossible to use this as incentive. The only way to do this would be to increase the overall efficiency of the permitting process, which is a worthwhile policy goal in itself.

5.3 Differing Standards of Quality for Inclusionary Housing

Some inclusionary zoning statutes allow developers to build the inclusionary housing at lower cost standards than market-rate housing. IZ housing can be built more densely, have fewer frills, or be built at a less desirable location. Neither land consumption nor housing quality are modeled in this analysis and doing so would greatly complicate the analysis. The next section is equipped to deal with spatial variation and it is possible to include an analysis of this aspect in a future version of this paper.

6 Urban Aggregates

To understand the impact of IZ on urban development requires being able to aggregate the individual behavior of all developers across time and space. This is doable by introducing a location variable into the rent function. In previous sections of the analysis, rents depend only on time and are determined by long-run growth. To distinguish individual parcels, I add a rent a distance variable *z*, to the rent function.

Suppose that the city is monocentric and that the proximity to the central business district (CBD) is the only characteristic that distinguishes parcels of land from one another. The marginal cost of transport is assumed to be constant, at τ dollars per unit of distance. The total transportation cost, $\tau \cdot z$, will be capitalized into the rent[14] such that the market rent, in the absence of IZ, can be expressed as

$$R(t,z) = R(t,0) - \tau \cdot z$$

Given the structure of the model, the city will expand outwards from the CBD in a continuous manner.[15] The land that is being developed is land that is located at the current urban-rural boundary, Z. Thus, the rent at the time of development in terms of the urban boundary at time *t* is defined as

$$R(T,z) = R(t,Z(t))$$

Add $\tau \cdot (Z(t) - z)$ to both sides of the expression and re-arrange for a definition of urban market rents in terms of the urban-rural boundary:

Equation 4

$$R(t,z) = R(T(z),z) + \tau(Z(t) - z).$$

The rent at time *t* is equal to the rent at the time of development, which is dependent on distance, plus a location premium. As the city grows, so does rent on urban

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[14] See Capozza and Helsley (1989) for a derivation of this rent function.

[15] Intuitively, the continuous development pattern occurs because the hurdle rent increases in time and decreases in distance. This is not always the case in dynamic models of land development. See Turnbull (1988) for a model similar to the one in this paper but where “leapfrog” patterns of development may occur.

land at all locations by the difference in the cost of transport from the urban-rural boundary.

6.1 City size, density, and housing when the ceiling is binding

City Size. Determining the city size can be accomplished by substituting $R(T,z) = R(t, Z(t))$ into the first-order condition for timing when the ceiling for density is binding, Equation 3. Doing this yields a condition for the rent at the urban boundary.

$$R(t, Z(t)) = r \cdot \frac{C((1+\beta)\cdot \bar{h})}{(1+\beta)\cdot \bar{h}} \cdot \frac{1}{(1-\alpha)}$$

Substituting the transportation costs into the above yields an expression for the urban-rural boundary:

Equation 5

$$Z(t) = \frac{1}{\tau} \cdot \left(R(t,0) - r \cdot \frac{C((1+\beta)\cdot \bar{h})}{(1+\beta)\cdot \bar{h}} \cdot \frac{1}{(1-\alpha)} \right)$$

For the binding density ceiling, the urban boundary will contract with the unit transportation cost; expand with the base rent and over time as rents grow; contract from an increase in the interest rate; contract with an increase of the density bonus; contracts with the severity of affordable housing requirements (set-aside and subsidy); and contracts as the density ceiling rises toward the optimal density.[16] All of these comparative static results are consistent with the previous results concerning timing.

Housing Stock. The stock of housing is determined by both density and city size. For the sake of simplicity I assume a linear city originating from the CBD.[17] The total housing stock, $H(t)$, is then

$$H(t) = \int_0^{Z(t)} h(z)dz$$

Density can vary over distance. In this particular type of dynamic model, how it varies depends upon the nature of rent growth.[18] For linear rent growth, the density of

[16] It is also apparent that the city boundary is unique.
[17] One goal of a post-conference revision of this paper is to model a two-dimensional circular city.
[18] For a discussion, see McFarlane (1999), page 421.

development is constant and for geometric rent growth, it increases over distance.[19] Assuming that the density constraint is binding, and that growth is linear, then the density of development is constant. The total housing stock is given by a simple expression:

Equation 6

$$H(t) = (1+\beta)\cdot \bar{h} \cdot Z(t)$$

Substituting the expression for the urban rural boundary yields a more elaborate definition of the housing stock:

$$H(t) = \frac{1}{\tau}\cdot\left((1+\beta)\cdot \bar{h}\cdot R(t,0) - r\cdot C\left((1+\beta)\cdot \bar{h}\right)\cdot\frac{1}{(1-\alpha)}\right)$$

The stock of housing increases over time as rents grow, decreases with the cost of transportation and the interest rate. Raising the share of units dedicated to IZ and reducing the rent on IZ units will reduce the total housing stock. The effect of the density bonus on the housing stock is not obvious from the above equation. The partial derivative of Equation 6 with respect to the density bonus is:

$$\frac{\partial H(t)}{\partial \beta} = \bar{h}\cdot Z(t) + (1+\beta)\cdot \bar{h}\cdot\frac{\partial Z(t)}{\partial \beta}$$

The first term is positive: as density increases, the housing stock increases by the amount of housing units allowed under the ceiling. The second term is negative: increasing the bonus delays investment and thus reduces city size. Substituting the expression of city-size (Equation 3) yields:

$$\frac{\partial H(t)}{\partial \beta} = \frac{\bar{h}}{\tau}\cdot\left(R(t,0) - \frac{rC'\left((1+\beta)\cdot \bar{h}\right)}{(1-\alpha)}\right)$$

Add in $R(t,Z(t)) - R(t,Z(t))$ to the bracketed term and substitute the first-order condition from Equation 3 to get:

$$\frac{\partial H(t)}{\partial \beta} = \bar{h}\cdot Z(t) + \frac{r\cdot \bar{h}}{\tau\cdot(1-\alpha)}\cdot\left[\frac{C\left((1+\beta)\cdot \bar{h}\right)}{(1+\beta)\cdot \bar{h}} - C'\left((1+\beta)\cdot \bar{h}\right)\right]$$

The effect of the density bonus on housing stock is ambiguous and depends on the size of the city. The term in square brackets is negative because, given the production function, average costs are less than marginal costs. The second additive term represents

[19] Only the first (and last) development is modeled in this analysis. In order to add more realism to these dynamic models, sequential investment, and thus a declining density gradient has been modeled by some authors. A good discussion of models of redevelopment can be found in Brueckner (2000).

$(1+\beta)\cdot\bar{h}\cdot\partial Z(t)/\partial\beta$. There is an initial retraction of the housing stock due to an increase in the bonus, but once the building resumes at a higher density, the effect of the density bonus on the total housing stock will eventually be positive. Schuetz, Meltzer, and Been (2007) find that the longer a program has been in place, the greater the output of affordable housing. They stress gaining administrative familiarity and the simple fact that it takes time to build. Here, I present an explanation that supports their latter one. This model suggests that there will also be an initial contraction of the housing supply.

6.2 Average Rent

A critical issue is whether renters would gain from inclusionary zoning. Are rents lower or higher as a result of inclusionary zoning? This question should be asked for the tenants of both market-rate and inclusionary. The average market rate across all units, AR^M, is defined by

$$AR^M(t)=\frac{1}{(1-\lambda)\cdot H(t)}\cdot\int_0^{Z(t)}(1-\lambda(z))\cdot h(z)\cdot R(t,z)dz$$

The set aside, λ, is assumed to be constant over space $\lambda(z)=\lambda$. When the ceiling is binding, I know that density is constant and equal to the ceiling plus the density bonus: $(1+\beta)\cdot\bar{h}$. Solving the above integral I find that

$$AR^M(t)=\frac{1}{2}\cdot\left[R(t,0)+r\cdot\frac{C((1+\beta)\cdot\bar{h})}{(1+\beta)\cdot\bar{h}}\cdot\frac{1}{1-\alpha}\right].$$

When transport costs are linear, the average market-rate rent is equal to the average of the rent at the CBD and the periphery. The average rent rises over time and increases with the interest rate. I find that inclusionary zoning increases the rent paid by tenants of non-inclusionary zoning housing. The control, by causing a delay, raises the rent at the time of development. The density bonus also raises the cost of development but has the effect of increasing density closer to the optimum.

Tenants of inclusionary housing pay a fraction, θ, of the market-rate rent. If this is the case, then the average rent paid by those living in inclusionary housing, AR^{IZ}, is given by $\theta\cdot AR^M(t)$ expressed as:

$$AR^{IZ}(t) = \frac{\theta}{2} \cdot \left[R(t,0) + r \cdot \frac{C\left(\left(1+\beta\right)\cdot \bar{h}\right)}{\left(1+\beta\right)\cdot \bar{h}} \cdot \frac{1}{1-\lambda\cdot\left(1-\theta\right)} \right]$$

(I expand the summary term α for clarity.) Average rents in inclusionary zoning will decline from an increase in the control but the reduction is not one-for-one because an increase in the stringency of affordability requirements will delay development and result in higher market rents.

The rent paid by the average tenant is the weighted average of the expressions for AR^{M} and AR^{IZ} and is given by: $AR(t) = \left(1-\lambda\right)\cdot AR^{M}(t) + \lambda \cdot AR^{IZ}(t)$ or

$$AR(t) = \frac{1}{2} \cdot \left[\left(1-\alpha\right)\cdot R(t,0) + r \cdot \frac{C\left(\left(1+\beta\right)\cdot \bar{h}\right)}{\left(1+\beta\right)\cdot \bar{h}} \right]$$

Although rents increase on tenants of market-rate housing, the rent paid by the average tenant will decline as the control rises. The density bonus will increase the average rent.

6.3 Average Price of Land

First, consider how price varies across space. Within the urban boundary the price of land is the price of developed land, if $z \le Z(t)$, then $P(t,z) = P^{d}(t,z)$. Outside the urban boundary, the price of urban land is determined by the price of vacant land, if $z > Z(t)$, then $P(t,z) = P^{v}(t,z)$. For an expression of the price of developed land: integrate Equation 1 by parts as shown in Section 4.1; substitute the definition rents in terms of the urban boundary; and substitute the first-order condition with respect to timing:

$$P^{d}(t,z) = C\left(\left(1+\beta\right)\cdot \bar{h}\right) + \frac{\left(1+\beta\right)\cdot \bar{h}\cdot\left(1-\alpha\right)}{r} \cdot \left[\tau \cdot \left(Z(t) - z\right) + G(t)\right]$$

The price of developed can be separated into three components: construction cost, a location premium, and a growth premium. How does IZ affect the price of vacant land. The bonus raises the construction cost and the multiplier on the location and growth premium. The affordable housing requirement lowers the multiplier on the location and growth premium. Both parameters delay development and thus reduce the location premium.

The average price of developed land in a linear city would be

$$AP^d(t,z) = C\left((1+\beta)\cdot \bar{h}\right) + (1+\beta)\cdot \bar{h}\cdot(1-\alpha)\cdot\left[\frac{\tau\cdot Z(t)/2}{r} + \frac{G(t)}{r}\right]$$

Raising the bonus will increase the structural premium and increase the density multiplier and so raise the average price of developed land. However, the average location premium will decline as a result.

$$\frac{\partial AP^d(t,z)}{\partial\beta} = C'\left((1+\beta)\cdot\bar{h}\right)\bar{h} + \frac{\bar{h}\cdot(1-\alpha)}{r}\cdot\left[\tau\cdot Z(t)/2 + G(t)\right] + \frac{(1+\beta)\cdot\bar{h}\cdot(1-\alpha)}{r}\cdot\left[\tau/2\cdot\partial Z(t)/\partial\beta\right]$$

This last term is negative but the entire term simplifies to

$$\frac{\partial AP^d(t,z)}{\partial\beta} = \frac{C'\left((1+\beta)\cdot\bar{h}\right)\bar{h}}{2} + \frac{C\left((1+\beta)\cdot\bar{h}\right)}{2\cdot(1+\beta)} + \frac{\bar{h}\cdot(1-\alpha)}{r}\cdot\left[\tau\cdot Z(t)/2 + G(t)\right]$$

Raising the density bonus will increase the average price of land.

Raising the affordable housing requirement will reduce the value of future growth and will delay development so that the impact on the price of developed land is negative.

7 Flexible Density

Another scenario is that the ceiling is not binding. This is possible when the optimal density is less than the maximum density ceiling, $\bar{h}$. When this is the case, density becomes a choice variable. The objective function of the developer is:

$$\max_{T,h} P^v(t) = h\cdot\int_T^\infty (1-\alpha)\cdot R(s)e^{-r(s-t)}ds - C(h)\cdot e^{-r(T-t)}$$

where $\alpha \equiv \lambda\cdot(1-\theta)$. The bonus term does not appear because, in this scenario, developers would not take advantage of the bonus. Here the analysis would be identical to a tax on urban rents. Previous work on the subject (McFarlane, 1999) in a similar model shows that, when there are no agricultural rents, then the control has no impact on the timing of the development but is likely to reduce the density of development.

The first-order condition with respect to timing is:

$$P_T = -h\cdot(1-\alpha)\cdot R(T) + rC(h) = 0$$

The second-order condition with respect to timing is

$$P_{TT} = -h \cdot (1-\alpha) \cdot R'(T) < 0$$

Growth is required for development to be optimal. The change in timing due to an increase in the restrictiveness of affordable housing requirements when density is fixed is given by the implicit function theorem:

$$\frac{\partial T}{\partial \alpha} = -\frac{P_{T\alpha}}{P_{TT}}$$

Given that

$$P_{T\alpha} = hR(T) > 0$$

Raising the depth of the subsidy or the required offset will delay development timing when density is fixed. The first-order condition with respect to density is

$$P_h = \int_T^\infty (1-\alpha) \cdot R(s) e^{-r(s-T)} ds - C'(h) = 0$$

The marginal revenue must equal the marginal cost. The second-order condition is

$$P_{hh} = -C''(h) < 0$$

Marginal costs are increasing in density. The change in density with respect to the affordable housing requirements when timing is fixed is given by

$$\frac{\partial h}{\partial \alpha} = -\frac{P_{h\alpha}}{P_{hh}}$$

Given that

$$P_{h\alpha} = -\int_T^\infty R(s) e^{-r(s-t)} ds < 0$$

We see that an increase in affordable housing requirements would lower density.

To understand how these two decisions interact, we need the following equations

$$P_{Th} = -(1-\alpha) \cdot R(T) + rC'(h) > 0$$

which we know is positive from the properties of the cost function (multiply the F.O.C. with respect to timing by *h*) and

$$P_{hT} = -(1-\alpha)\cdot R(T) + r\int_{T}^{\infty}(1-\alpha)\cdot R(s)e^{-r(s-T)}ds = (1-\alpha)\cdot G(T) > 0$$

The effect on timing of the affordable housing requirements is given by

$$\partial T/\partial\alpha = J^{-1}\cdot\left(P_{Th}P_{h\alpha} - P_{T\alpha}P_{hh}\right)$$

This result is ambiguous without further inspection. Substituting the F.O.C. with respect to timing gives us

$$\partial T/\partial\alpha = J^{-1}\cdot\frac{r}{(1-\alpha)}\cdot\left(C'(h)\cdot\left(C(h)/h - C'(h)\right) - C(h)\cdot C''(h)\right)$$

Assuming the Cobb-Douglas production function, $C(h)=h^{1/\gamma}$, there is no impact on timing. This result only makes sense in light of the comparative statics with respect to density. The Jacobian, which is positive by assumption, is given by

$$J = (1/\gamma - 1)\cdot h^{1/\gamma-1}\left(\alpha/\gamma\cdot R'(T) - (1-\alpha)\cdot G(T)\cdot r\right) > 0$$

The effect on density is given by

$$\partial h/\partial\alpha = J^{-1}\cdot\left(P_{T\alpha}P_{hT} - P_{h\alpha}P_{TT}\right)$$

Again this is not obvious without further inspection. Insert the FOC with respect to timing:

$$\partial h/\partial\alpha = J^{-1}\cdot\frac{h\cdot G(T)}{1-\gamma}\cdot\left((1-\alpha)\cdot\gamma\cdot G(T) - \alpha\cdot\left(\frac{R'(T)}{r}\right)\right)$$

Density falls because of the decline in revenue. Inclusionary zoning creates two countervailing effects on the timing decision. The first one is to delay. This first is a direct effect by raising the hurdle rent needed to cover costs. The second one is to hasten development. The second is indirect through a reduction of costs by a lowering of density. Under the current assumptions, these two effects offset one another so that there is no effect on timing. The housing supply will be reduced through the reduction of density of density but this may not be observed in the pace of land development. In addition, because the timing of development would not be altered, the rents paid by market-rate units would not rise.

8 Summary and Extensions

I have presented a standard model of the optimal timing of the development of urban land and asked how inclusionary zoning would affect the decision to invest. I have found that, in a regime where the density of development is fixed by regulation at a suboptimal level, that:

- Increasing the share of affordable housing will delay development;
- Increasing the restrictiveness of income targets for inclusionary housing will delay development;
- Increasing the density bonus will delay development;
- Providing a fee waiver and fast-tracking development will hasten development;
- When participation is voluntary, the density bonus can offset the reduction in rent revenue.

If density of development is flexible, then the impact of IZ is to reduce density. There will be no impact on the timing of development in the case of linear growth/

I then integrated a location variable into this model to develop a long-run model of the supply of housing in the presence of an exogenously growing demand. I found the following impacts of inclusionary zoning on the urban area (when the density of development is restricted):

- The urban boundary will contract with the density bonus and the strictness of the rent affordability requirements.
- Raising the proportion of IZ units and lowering the rent on IZ units will lower the total housing stock.
- Raising the density bonus has an ambiguous effect on the stock of housing and over time, it will expand the housing stock.

- The average rent of market-rate housing will rise by increasing the proportional rent reduction due to affordable housing provisions;
- The average rent of inclusionary housing and of all housing will decrease by increasing the proportional rent reduction due to affordable housing provisions.

These results are in agreement with the basic point made by opponents of inclusionary zoning, that it will restrict supply and raise prices. However, this is not because inclusionary zoning forces developers to other or outlying areas. Instead, there is a delay of development. Outlying areas with different regulations would develop comparatively early as a result of IZ close to the CBD but the actual time of development in outlying areas with a different regulatory structure would not change.

Extensions This analysis has been by no means exhaustive. The implementation of inclusionary zoning varies considerably and in this paper, the author has developed a framework for analyzing many of these complexities. There remain some refinements of the basic model. It is necessary to model the rent subsidy in a more transparent fashion. While it makes sense to express the subsidy as a percentage of rents, this formulation may be misleading in the spatial analysis. The current analysis was written with rental housing in mind. Adapting it for owner-occupied housing by including policies explicitly aimed at owner-occupied housing such as resale controls is a next step.

Consider the incentives. The density bonus has been assumed throughout this article to apply equally to affordable housing and to market rate development. However, this is not always the case. Jurisdictions have codes where the density bonus applies only to affordable housing. A density bonus can also vary by the size of the development and the location of development. All of these variations can be analyzed in the present model. There are other types of incentives not touched upon here: loans offered, grants, tax credits, relaxing development standards, and allowing construction of IZ units off-site.

The type of control can also vary. Raising the share of affordable units can increase the density bonus. There may also be the possibility of receiving additional cost offsets. I have assumed that rents will be regulated for perpetuity. This is the case for many of the jurisdictions studied by Schuetz et al. (2008): one-fifth in San Francisco and one-third in the Boston suburbs. There are other inclusionary regulations that allow affordable housing units to return to a market rent after a specific period of time, as short as ten years. I have also assumed throughout the analysis that affordable housing rents are a proportion of the current market rent. This is likely to be the case because eligibility requirements are determined by income. However, it is worthwhile examining to what extent jurisdictions regulate the rent on inclusionary units. If not, then there is a second and more important question worth investigating: whether rents in inclusionary units are significantly lower than market rate units? Lower income individuals may be willing to pay a significant premium to live in an advantageous location. This model, however, is not sufficiently elaborate to answer these questions.

A final aspect of the IZ policy that has to be analyzed is the option of the cash contribution, which is fairly standard.[20] In this, developers have the option of participating or making a payment to the local government. The effect of this payment can be analyzed fairly easily using the framework used to understand the impact fee. Understanding which choice developers make and how it depends on the incentives offered could be arrived at through the methods used to understand voluntary participation.

There are complex but doable extensions using this model. First, one could also examine to what extent results depend on the cost function. Second, it is likely that the first development may not be constrained by the maximum density requirement but that future ones would. The insights into the development of high density housing may hold but the insights into the initial development would be revealing. There are other investigations that are worthwhile but would require significant new efforts. One could model the demand-side in a more elaborate fashion, which may require a different type of model than the one presented here. There is also the possibility of modeling the benefits of all regulations, the density ceiling included. Such an exercise would be considerably more complex, however, there are guidelines on how to do this from previous work. In

[20] According to the Center for Housing Policy (2008), 86 percent of the jurisdictions in the San Francisco allow developers to pay fees in lieu of building units, only 38 percent in suburban Boston follow this practice; but 100 percent of all jurisdictions in the Washington DC area do.

addition, modeling the spatial variation of regulatory regimes could reveal more on whether inclusionary zoning itself could contribute to urban sprawl.

Lessons. It is no surprise that an affordable housing requirement imposed on builders would create a distortion in the housing market leading to higher rents for those not in inclusionary housing. This analysis highlights exactly how this happens and how it can be counterbalanced through incentives that reduce the costs of development. It is not, however, the purpose of this paper to condemn IZ as a housing policy. Equity is a policy goal that cannot be properly addressed in the theoretical framework presented in this paper. But planners should be aware that inclusionary zoning has redistributive properties through its impact on the housing market and should think about avoiding the more distortionary effects. This analysis ignores any efficiency from providing affordable housing, which may offset some of the deadweight loss. There are significant benefits to be gained from the provision of affordable housing, especially if it is for local government employees who need to be compensated less as a result of the housing and are able to perform their jobs more effectively. There is, however, an inherent contradiction in making efficiency arguments for inclusionary zoning because IZ only results in an increase in housing supply when there is a binding density ceiling, a ceiling which itself is often motivated by efficiency arguments. An alternative affordable housing policy would be to raise the permissible density of all developments.

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