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AN ANALYSIS OF ALTERNATIVE MEASURES OF TENANT BENEFITS OF GOVERNMENT HOUSING PROGRAMS WITH ILLUSTRATIVE CALCULATIONS FROM PUBLIC HOUSING

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One means of conducting a benefit-cost analysis of a government program is to estimate the net benefit of the program to each group of similarly affected people. These net benefits are added to obtain the total net benefit of the program. An important group of people affected by government housing programs is the people who occupy housing directly affected by the programs. The primary purposes of this paper are to derive a conceptually meaningful measure of net tenant benefit applicable to any government housing program, to state the properties of several alternative measures vis-a-vis this measure, and to show how much the estimates yielded by these alternative measures differ from the estimate yielded by the best measure for a sample of individual data from the federal public housing program. In the process, we will provide a rough idea of the magnitude of mean tenant benefit in public housing, the percentage increase in the consumption of housing service experienced by

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loses) because he spends less (or more) money on housing service and, hence, is able to consume more (or less) non-housing goods. The value of this component of the net benefit is simply $P_m Q_m - P_c Q_c$, the excess of before-program housing expenditure over after-program housing expenditure. Second, the tenant benefits (or loses) because he consumes more (or less) housing service. By definition, the height of the demand curve at any quantity is the value that the consumer places on an infinitesimally small increase in the quantity of housing service consumed. Consequently, the value that the consumer places on consuming Q_c rather than Q_m units of housing service per time period is $\int_{Q_m}^{Q_c} D$, the area under the demand curve between Q_m and Q_c.[3] The net benefit to the tenant per time period is the sum of these two components.

$$(1) \qquad B = P_m Q_m - P_c Q_c + \int_{Q_m}^{Q_c} D$$

This expression is easily shown to be the excess of consumer's surplus at (Q_c, P_c) over the consumer's surplus at (Q_m, P_m).[4] Hence, an estimate of B is an estimate of the maximum amount of money that the tenant would pay during the time period (in addition to $P_c Q_c$) in order to receive the benefits of the program if his best alternative is to purchase housing service on the private market.[5]

The expression (1) would be of little use to benefit-cost analysis in the absence of a demand curve for housing service. Fortunately, Richard Muth's extensive statistical analysis [3] suggests a unitary price elasticity of demand for housing service. Therefore, the best statistical evidence suggests individual demand curves of the form, $P = P_m Q_m / Q$.[6] Hence, the best practical measure of net benefit to an individual tenant appears to be

(2) $$B = P_m Q_m - P_c Q_c + P_m Q_m (\ln P_m Q_c - \ln P_m Q_m).$$ [7]

We are interested in total tenant benefits for each program rather than the benefit to any particular tenant. Different tenants will place different values on a housing program for many reasons. They live in different housing markets where different prices of housing service prevail. They face different prices for non-housing goods, have different incomes and tastes, and, hence, they will demand different quantities of housing service at the same price. Because of the structure of the housing program, different tenants will occupy dwelling units emitting different quantities of housing service and will pay different rents. With the subscript <u>i</u> denoting the <u>i</u>th tenant directly affected by the particular housing program, the best practical measure of total net benefit to all tenants is

(3) $$B_T = \sum_{i=1}^{n} \left[(P_m Q_m)_i - (P_c Q_c)_i + (P_m Q_m)_i \left(\ln (P_m Q_c)_i - \ln (P_m Q_m)_i\right) \right]$$

This is simply the sum of individual net benefits as measured by formula (2)

II. Alternative Measures

The measure of tenant benefit derived in section I has a clear conceptual meaning. It is consistent with the theory of consumer choice and the best available empirical work on the demand curve for housing service. To our knowledge, this measure has never been used to obtain tenant benefits of a housing program. Instead, measures requiring less information and calculation have been used. These measures can be viewed as approximations to the best measure. In section III we will show how good these approximations are for a small sample of public housing tenants. In this section, the alternative measures and some of their properties will be stated.

Prescott [6] measured individual tenant benefit in public housing as the excess of the market rent of a public housing unit over the rent paid by its tenant. Hence, his measure of total tenant benefits is simply

$$(4) \qquad B_T^P = \sum_{i=1}^{n} [(P_m Q_c)_i - (P_c Q_c)_i] = \sum_{i=1}^{n} (P_m Q_c)_i - \sum_{i=1}^{n} (P_c Q_c)_i \ .$$

In obtaining a distribution of tenant benefits by income from Prescott's data, Robert Bish [2] accepted this measure. The Prescott measure of individual tenant benefit differs from the best measure by $P_m (Q_c - Q_m) - \int_{Q_m}^{Q_c} D$, and, hence, it approximates the measure (2) to the extent that $P_m (Q_c - Q_m)$ approximates $\int_{Q_m}^{Q_c} D$. In Figure 1, the error involved in using the Prescott measure is represented by the shaded area. The Prescott measure produces a larger estimate of benefit to each tenant than the best measure because $P_m (Q_c - Q_m) \geq \int_{Q_m}^{Q_c} D$ [8] for $D' < 0$. Hence, the Prescott measure always overestimates total net benefits.[9]

Olsen [4] measured mean tenant benefits in public housing by substituting estimates of the mean values of $P_m Q_c$, $P_m Q_m$, and $P_c Q_c$ into formula (2). This is equivalent to using a consumers' surplus measure of total tenant benefits instead of the sum of consumer's surpluses. Hence, his measure of total tenant benefits is

$$(5) \quad B_T^O = \sum_{i=1}^{n} (P_m Q_m)_i - \sum_{i=1}^{n} (P_c Q_c)_i + [\sum_{i=1}^{n} (P_m Q_m)_i] [\ln \sum_{i=1}^{n} (P_m Q_c)_i - \ln \sum_{i=1}^{n} (P_m Q_m)_i]$$

This measure may give either larger or smaller estimates of tenant benefits than the best measure.

Despairing of accounting for the difference between the "quality" of housing occupied by the tenant under a government program and on the private market, some might be tempted to omit the last term in expression (2) and to measure the benefit to a tenant by the reduction in his housing expenditure. In this case, total tenant benefits would be measured by

$$(6) \qquad B_T^X = \sum_{i=1}^{n} \left[(P_m Q_m)_i - (P_c Q_c)_i \right] = \sum_{i=1}^{n} (P_m Q_m)_i - \sum_{i=1}^{n} (P_c Q_c)_i$$

For programs which result in better housing for tenants, this expression clearly underestimates true tenant benefits. For programs which result in worse housing, it overestimates true benefits.

The virtue of the alternative measures discussed in this section is that they are less costly to use than the best measure. The same purpose could be achieved by calculating individual tenant benefits by formula (2) for k of the n families directly affected by the program. Total tenant benefits could be measured by

$$(7) \qquad B_T^S = (n/k) \sum_{i=1} \left[(P_m Q_m)_i - (P_c Q_c)i + (P_m Q_m)_i \left(\ln (P_m Q_c)_i - \ln (P_m Q_m)_i \right) \right].$$

If the k families are a random sample of the total population, then B_T^S is an unbiased estimator of B_T.

III. Estimation and Analysis of Tenant Benefits in Public Housing

The data are for 1960 and include 498 observations on publicly housed families in the six states of Alabama, New Jersey, North Dakota, New York, Virginia, and California.[10] The projects range in scale from 40 to 300 dwelling units and are located both in large metropolitan areas and smaller rural communities.

P_mQ_c is estimated for each tenant family from admission income limits for the individual projects in the manner described in [6, 7]. As P_mQ_c is required by law to exceed P_cQ_c by 25 percent for families with incomes at the admission limit and P_cQ_c is 20 percent of the tenant's money income, limits for admission may be used to estimate the "private equivalent" rental.[11] The admission limits (and hence P_mQ_c) are differentiated for each project by family size classes.

For project 1, P_mQ_m is known from personal interviews; for projects 2-6, Census data [8] are used to estimate housing expenditure functions and these estimated housing expenditure functions are used to predict P_mQ_m for public housing tenants.[12] To check the validity of this procedure income elasticities estimated from the tenant data for project 1 are compared to similar estimates using pooled Census observations for six urbanized areas in Alabama. (The Census observations were restricted to annual incomes equal to or below $3,500, the highest family income in project 1). Both the arithmetic and double-log income elasticities from the tenant data (.316 and .315 respectively) are comparable to the double-log estimate of .321 using the pooled observations. For projects 2-6 similarly pooled Census data for the

2 urbanized areas nearest to the project were used to predict $P_m Q_m$. These income elasticities are somewhat lower ranging from .101 to .296. The estimates of $P_c Q_c$ and family income are obtained directly from the project's annual reexamination report.

Table 1 shows the results of the calculations for the six projects and for all 498 tenant observations combined (row 7). $\overline{B}$ is the mean net monthly benefit estimated by the methods discussed above; the percentage errors compare the benefit estimate with B_T. Both B_T^P and B_T^O consistently overestimate B_T for all projects and the combined sample; B_T^O is also consistently closer than B_T^P to the best estimate. The B_T^P and B_T^O errors for all observations are about 6.4 and 1.8 percent respectively. As expected B_T^X seriously underestimates tenant benefits by about 39 percent for the entire sample of publicly housed families and can be eliminated from further consideration. To estimate B_T^S a 25 percent randomly selected sample of families was taken. The combined sample net benefit figure overestimates B_T by only 1.7 percent; other than sampling variability there is no apparent reason for B_T^S to under or overestimate B_T.

These results suggest the following conclusions regarding the empirical estimation of net tenant benefits in public housing. Of the measures considered, B_T^O is the best estimate of B_T since it produces (with one exception) the lowest percentage errors of all four methods. Our data suggest that aggregated project data may, therefore, be used confidently both with regard to inter-project comparisons and program evaluation. B_T^P produces quite reliable estimates while requiring less information (i.e., it is not necessary to predict $P_m Q_m$). Furthermore,

Table 1

ALTERNATIVE ESTIMATES OF MEAN TENANT (MONTHLY)
BENEFITS AND PERCENTAGE ERRORS

		B_T	B_T^P	B_T^O	B_T^X	B_T^S
(1)	$\bar{B}$	30.675	37.475	33.625	15.750	29.275
	% error	-	.2217	.0962	-.4866	-.0456
(2)	$\bar{B}$	38.375	40.250	38.550	22.800	32.900
	% error	-	.0489	.0046	-.4059	-.1427
(3)	$\bar{B}$	21.600	22.483	21.750	12.733	19.667
	% error	-	.0409	.0069	-.4105	-.0895
(4)	$\bar{B}$	24.690	27.225	25.521	11.718	27.437
	% error	-	.1027	.0337	-.5254	.1113
(5)	$\bar{B}$	22.990	23.396	23.271	19.250	26.542
	% error	--	.0177	.0122	-.1627	.1545
(6)	$\bar{B}$	25.073	26.120	25.094	14.901	25.461
	% error	-	.0418	.0008	-.4057	.0155
(7)	Total (1-6)					
	$\bar{B}$	25.717	27.361	26.167	15.727	26.157
	% error	-	.0639	.0175	-.3885	.0171

Source: These calculations are based on unpublished records of the U. S. Public Housing Administration and published data of the U. S. Bureau of the Census [8].

for our sample it ranked projects as accurately as B_T^O. B_T^S is an essentially different way of estimating tenant benefits since it requires data on individual families rather than average project data. It may produce wide disparities in percentage errors among projects though the cancelling of these errors over projects results in a very accurate estimate of net tenant benefits. Though the number of projects is small, it should be noted that the sum of positive rank differences taken with respect to the ordering of monthly benefits by B_T is smaller for B_T^O and B_T^P than for B_T^S.

There are several other inferences about public housing which can be drawn from our sample. Public housing permits its tenants to consume more non-housing goods as well as more housing service. The positive value for B_T^X indicates that the average public housing tenant spends less on housing than he would have spent on the private market in the absence of a subsidy.[13] Hence, he consumes a greater quantity of non-housing goods than otherwise. For our sample, more than 60 percent [$15.73/$25.72] of the benefit of public housing to its tenants is due to the greater quantity of non-housing goods consumed.

Public housing tenants also benefit because they consume more housing service. We estimate that 93 percent of our sample occupied better housing as a result of the public housing program. If the assumptions made thus far are valid and if all public housing units of the same size in our sample emit the same quantity of housing service per time period, then we could say that the public housing tenants in our sample occupied dwelling units emitting 18 percent more housing service than they would have consumed in the absence of public housing.[14] This accounts for somewhat less than 40 percent of the benefits of public housing to its tenants. Therefore, it appears that most of the benefits of public housing to its tenants are not due to greater consumption of housing service.

Since the mean monthly income of tenants in our sample was $249.22, public housing has increased the real income of our sample by about 10 percent [$25.72/$249.22].

Finally, we consider how tenant benefits in federal public housing vary among families with different incomes and sizes. The benefit to a family depends on its expenditure on housing in the absence of a housing subsidy, the amount of rent that it must pay for its public housing unit, and the market rent of this unit. The first two variables vary directly with income. The third varies directly with family size. Hence, we expect tenant benefit to vary systematically with family income and size. As a compact way of presenting this variation, we estimated the following statistical relationship by the method of least squares for the 498 families in the 6 projects:

$$B = 47.46 - .12Y + 1.80S \qquad R^2 = .55;\ F = 308.39$$
$$\qquad (-24.46)\quad (7.38)$$

where

B = the estimated monthly value of tenant benefit using the best practical measure

Y = the family's monthly income

S = the number of persons in the family.

The numbers in parentheses are the t-scores. If the error term in the true stochastic relationship satisfies the Gauss-Markov conditions and is normally distributed, then we would accept the hypotheses that tenant benefit varies inversely with family income and directly with family size at much less than the one percent level of significance.[15] The same results were obtained for each project separately.

The predictions of P_mQ_m and P_mQ_c in this paper are very rough. However, we believe that reliable predictions of these magnitudes are possible in the case of many housing programs using information already collected by the U.S. Department of Housing and Urban Development and by the U.S. Bureau of the Census. The latter agency has individual data on housing expenditure, income, family size, and other family characteristics. For families in a housing market, housing expenditures could be regressed on family characteristics, and the estimated equation used to predict the private market housing expenditures of families living in that market who are direct beneficiaries of a housing program. The Census also has individual data on the characteristics of housing units. The rent of housing units in each market could be regressed on housing characteristics, and the estimated equation used to predict the rent which could be obtained on the private market for dwellings rented at below-market prices under the government program. HUD probably has sufficient information about the characteristics of the families occupying public housing and the characteristics of the units that they occupy to make reliable estimates for this program. If the Census were to report mean values of housing and family characteristics for urban places and for Census tracts in SMSA's and if HUD were to make information on the mean values of these characteristics for each project or locality accessible, then it would be feasible for nongovernment researchers to make reliable estimates of tenant benefits of government housing programs.

IV. Conclusions

Because the sample is small and nonrandom and the methods used to predict housing expenditures and market rents are crude, all of the results of this paper should be considered tentative. However, our sample seems to indicate the following conclusions with respect to public housing.

1. Good estimates of tenant benefits are possible using mean data for projects or for the program as a whole.
2. In 1960, the mean tenant benefit in public housing was in the neighborhood of $300.
3. Public housing results in roughly a 10 percent increase in the real income of its tenants.
4. On the average, public housing tenants consume 18 percent more housing service than they would have consumed in the absence of a housing subsidy.
5. About 60 percent of the benefit of public housing to its tenants is attributable to greater consumption of nonhousing goods and 40 percent attributable to greater consumption of housing service.
6. Tenant benefit in public housing varies inversely with family income and directly with family size.

The results of this paper are based on rough methods of predicting the market values of public housing units and the housing expenditure of their occupants in the absence of a housing subsidy. Better methods of making these predictions deserve a high priority in the field of benefit-cost analysis applied to government housing programs.

Appendix

Public housing tenants typically consume a different quantity of housing service than they would have consumed in the absence of a housing subsidy. In this appendix, we will show that this difference in percentage terms may be predicted for a set of public housing tenants living in different housing markets (where the price of housing service is not necessarily the same) using predictions of the market rents of the public housing units occupied by these tenants and the housing expenditures of these families in the absence of a housing subsidy and a knowledge of the upper income limit for admission in the various projects for families of any one size.

We assume that the housing market in which each of the projects is located is in short-run equilibrium so that there is only one price per unit of housing service prevailing in each of these housing markets. We denote the price per unit of housing service in the *j*th market by P_j. If the process of setting upper income limits for admission is as we have described, then

(A.1) $$P_j Q_{ij} = (1/4) U_{kj}$$

for all apartments occupied by families with *k* members where Q_{ij} is the quantity of housing service per time period emitted by the apartment occupied by the *i*th family in the project located in the *j*th market and U_{kj} is the upper income limit for admission applicable to a family with *k* members living in the project in the *j*th housing market.

Finally, we assume that families with *k* members consume the same quantity of housing service regardless of the project in which they live. In other words,

(A.2) $$Q_{ij} = Q_k$$

for all apartments occupied by families with _k_ members. This assumption seems rather plausible for two reasons. First, we believe that there is little variation in the number of rooms assigned to families of a particular size in public housing. Public housing administrators throughout the country have come to accept a similar notion of how many rooms a family of _k_ persons "needs." Second, all six of the projects in our sample were built at about the same time (1959-1960). Public housing construction appears to be highly standardized. Hence, to use an old distinction in housing economics, all families with _k_ members in our sample occupy housing of approximately the same quantity and quality. In our terms, they all consume about the same quantity of housing service. (A.1) and (A.2) imply that $P_j = (1/4Q_k)U_{kj}$ and, hence, that

(A.3) $$\sum_{j=1}^{6}\sum_{i=1}^{n_j}(Q_{ij} - Q^*_{ij}) / \sum_{j=1}^{6}\sum_{i=1}^{n_j} Q^*_{ij}$$

$$= \sum_{j=1}^{6}\sum_{i=1}^{n_j}[(P_jQ_{ij}/U_{kj}) - (P_jQ^*_{ij}/U_{kj})]/\sum_{j=1}^{6}\sum_{i=1}^{n_j}(P_jQ^*_{ij}/U_{kj})]$$

where Q^*_{ij} is the quantity of housing service which the _i_th family in the _j_th project would consume in the absence of a housing subsidy and n_j is the number of families in the _j_th project. Since we have predictions of P_jQ_{ij} and $P_jQ^*_{ij}$ for each family in our sample and since we know U_{kj} for all projects and family sizes, we can estimate the percentage increase in consumption of housing service for our sample of public housing tenants. If our assumptions are correct, then we would get the same answer regardless of the value of k used. We calculated the right-hand side of (A.3) for family sizes 1 through 6. The largest value obtained is .180 and the smallest value .175. The small variation in these results supports the validity of our assumptions and leads us to infer that the public housing tenants in our sample consume about 18 percent more housing service than they would have consumed in the absence of a housing subsidy.

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Footnotes

[1]This theory of the housing market is rigorously developed in [3] and [5].

[2]Notice that (Q_c, P_c) is not a point on the tenant's demand curve. This is typical of government housing programs. For example, see Prescott [7].

[3]Notice that if $Q_c < Q_m$, then $\int_{Q_m}^{Q_c} D < 0$ as it should be.

[4]The consumer's surplus at a point (Q,P) is $(\int_o^Q D) - PQ$. Denote the excess of consumer's surplus at (Q_c, P_c) over consumer's surplus at (Q_m, P_m) by E. Then, $E = [\int_o^{Q_c} D) - P_cQ_c] - [(\int_o^{Q_m} D) - P_mQ_m] = P_mQ_m - P_cQ_c + [(\int_o^{Q_c} D) - (\int_o^{Q_m} D)] = P_mQ_m - P_cQ_c + \int_{Q_m}^{Q_c} D = B.$

[5]This much is known by many economists. Indeed, a correct intuitive discussion of this result appears in a prominent principles textbook [1, pp. 136-37]. However, to our knowledge this measure has never been stated, derived, or used by any writer in the field of housing.

[6]Of course, we realize that Muth's finding of unitary price elasticity for an aggregate demand curve does not imply that each individual's demand curve has unitary price elasticity. Indeed, we expect otherwise. Consequently, our estimate of the benefit to an individual tenant will err for this reason. We foresee little hope of eliminating this source of error. At least, it can be said that our assumption about individual demand curves is consistent with Muth's finding. Aside from this issue, additional confirmation of Muth's result would naturally make us more confident in our findings.

[7] $\int_{Q_m}^{Q_c} D = \int_{Q_m}^{Q_c} P_mQ_m/Q = P_mQ_m [\ln Q_c - \ln Q_m] = P_mQ_m [\ln P_mQ_c - \ln P_mQ_m].$

[8]This inequality holds for $Q_c < Q_m$ as well as for $Q_c \geq Q_m$.

[9]In effect, Prescott valued the increase in the consumption of housing service by the tenant at market price while the tenant places a lower value on such increases.

[10]The project names are coded on the request of the federal public housing authorities. The authors will provide the input data for all six projects to interested readers on request.

[11]The relevant provisions of the law are sections (2) (1) and (15) (7) (b) (ii) of the United States Housing Act of 1937. See [9]. These provisions lead us to predict a market rent for each unit equal to one-fourth of the upper income limit applicable to families of the size eligible for the unit. We are not at all certain the extent to which these somewhat vague provisions are adhered to by local public housing authorities. A better method of predicting $P_m Q_c$ is of utmost importance to making further advances in benefit-cost analysis in this area.

[12]This source gives median gross rent by income class. Our data are median rents and the midpoints of the corresponding income classes.

[13]B^x is positive for 92.8 percent of our sample.

[14]See the appendix for a rigorous derivation of this result. There is quite a variation in this percentage among projects ranging from 7 percent in project 5 to 46 percent in project 1.

[15]Using Prescott's data and measure of tenant benefit, Bish [2] has computed mean tenant benefit by income class. He found that higher income classes had smaller mean benefits.