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DYNAMICS OF PARTICIPATION IN A HOUSING ALLOWANCE PROGRAM

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PREFACE

This note was prepared for the Office of Policy Development and Research, U.S. Department of Housing and Urban Development. It formulates a dynamic model of participation in the Brown and St. Joseph County housing allowance programs, estimates the model's parameters from pooled data for the two sites, and uses the fitted model to estimate the equilibrium level of enrollment and the time required to reach it. Although the model is a considerable simplification of reality, it shows how current enrollment depends on the underlying dynamics of eligibility changes, explains why enrollment is lower than many people expected, and clarifies the prospects for larger enrollment.

The administrative records of the experimental allowance program are extraordinarily rich in their details of client characteristics and transactions with participants. They will support more complex models and more detailed analysis than are reported here. The goal of further research should be to add precision and detail without losing the methodological power of this first-generation model.

The model reported here was devised by C. Peter Rydell. John E. Mulford helped to develop its details and Lawrence Kozimor helped to fit its parameters. Much of the data was drawn from Kozimor's *Two Years of Housing Allowances: Eligibility and Participation*, The Rand Corporation, WN-9816-HUD, forthcoming. The HAO administrative records were prepared by the staffs of the housing allowance offices in Brown and St. Joseph counties and reorganized into research files by Iao Katagiri and Ann Wang.

Drafts of this note were reviewed by James R. Hosek and Ira S. Lowry. Judy Bartulski and Ned Harcum prepared the draft typescript and tables. Production typist was Joan Pederson. Charlotte Cox edited the report and supervised its publication.

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SUMMARY

Three years after the housing allowance program began in Brown County, Wisconsin, and St. Joseph County, Indiana, the participation rate of eligible households was 40 percent. Analysis of participation dynamics predicts that when enrollment reaches equilibrium, the participation rate will be about 51 percent. It also indicates that enrollment will reach 95 percent of the equilibrium level about 5.5 years from the start of the program.

Not all eligible households will participate in the program--even when enrollment reaches equilibrium--because the eligible population is continuously turning over and enrollment is not instantaneous. There will always be households that became eligible too recently to have yet joined the program.

In general, the equilibrium participation rate equals the enrollment rate (fraction of nonenrolled eligibles that join the program in a year) divided by the sum of the enrollment rate and the termination rate (fraction of participants that leave the program in a year). For nonelderly households, the annual enrollment rate is 51 percent and the termination rate is 45 percent, making their equilibrium participation rate 53 percent. For elderly households, the annual enrollment rate is 20 percent and the termination rate is 21 percent, making their equilibrium participation rate 49 percent. Because eligible households in Brown and St. Joseph counties are half nonelderly and half elderly, the overall equilibrium participation rate is 51 percent.

The 51 percent equilibrium participation rate may strike some observers as surprisingly low. In fact, it is not low compared with other government transfer programs. For example, the equilibrium participation rate for the welfare program in New York City is 56 percent, as the comparison below shows:

Type of Household	Annual Enrollment Rate (%)	Annual Termination Rate (%)	Equilibrium Participation Rate (%)
Housing Allowance Program, 1977			
Nonelderly	51	45	53
Elderly	20	21	49
All	35	32	51
New York City Welfare, 1970			
Single-parent	248	19	93
Elderly	11	23	32
Other	58	66	47
All	49	35	56

To increase the equilibrium participation rate, one must either raise the enrollment rate or lower the termination rate. The equilibrium participation rate would be 100 percent only if the termination rate were zero--that is, only if there were a permanent group of participants.

Note, however, that high equilibrium participation rates in housing allowance or welfare programs are caused by high enrollment rates rather than low termination rates. For example, single-parent welfare cases have a 93 percent equilibrium participation rate because they have a 248 percent annual enrollment rate.

It is not suggested that enrollment rates in the housing allowance program could be increased to the high rates for single-parent welfare cases. However, they might be raised by as much as half (from 51 to 76 percent for nonelderly households and from 20 to 30 percent for elderly households), which would raise the equilibrium participation rate for the housing allowance program from 51 to 61 percent.

The termination rate is the sum of the rates at which eligible households escape poverty, become ineligible by moving to other assistance programs or changing their household composition, leave by outmigration or death, or leave the program while remaining eligible. Advocates of higher participation rates would hardly recommend that they be achieved by decreasing the rate of escape from poverty, or by allowing double assistance. Rates of migration, death, or household change are not controllable by allowance program managers. The

remaining method of increasing participation rates is to reduce the rate at which participants leave the program even though they are still eligible. That rate is 5 percent per year for nonelderly households and 4 percent per year for elderly households. However, even if those rates could be reduced to zero, the overall equilibrium participation rate would only increase from 51 to 55 percent.

CONTENTS

TABLES

I. INTRODUCTION

Eligibility and participation in the experimental housing allowance program in Brown County, Wisconsin, and St. Joseph County, Indiana, vary greatly by type of household. As shown in Table 1, among the four-fifths of the population that is nonelderly, only 12 percent are eligible; but 45 percent of those eligible participate in the program.

Table 1

PARTICIPATION IN THE HOUSING ALLOWANCE PROGRAM AT THE END OF YEAR 3: JUNE 1977 IN BROWN COUNTY AND DECEMBER 1977 IN ST. JOSEPH COUNTY

Type of Household	Number of Households			Eligibility Rate[a] (%)	Participation Rate[b] (%)
	All	Eligible	Enrolled		
Brown County					
Nonelderly	36,500	4,600	2,050	13	45
Elderly	7,300	3,400	1,350	47	40
All	43,800	8,000	3,400	18	43
St. Joseph County					
Nonelderly	59,800	7,200	3,270	12	45
Elderly	14,500	8,400	2,860	58	34
All	74,300	15,600	6,130	21	39
Both Counties					
Nonelderly	96,300	11,800	5,320	12	45
Elderly	21,800	11,800	4,210	54	36
All	118,100	23,600	9,530	20	40

SOURCE: Housing allowance office administrative records for year 3 and HASE survey of households at baseline, Sites I and II.

NOTE: Entries in the first two columns (all households, eligible households) are estimates from baseline surveys conducted just before enrollment began. Thus, the eligibility and participation rates shown in the last two columns do not reflect possible changes in the number of households or the number eligible during the first three years of enrollment.

[a]Eligibles as percent of all households.

[b]Enrolled as percent of all eligibles.

In contrast, among the one-fifth of the population that is elderly, 54 percent are eligible; but only 36 percent of those eligible participate in the program. The overall participation rate (three years after the program began) is 40 percent.

This note offers a dynamic explanation of the participation rate.* It analyzes the participation of nonelderly and elderly households in parallel and then combines the results into the following explanation of why the participation rate is less than 100 percent:

- First, the enrollment process has not yet reached equilibrium; the predicted equilibrium participation rate is 51 percent.
- Second, the 49 percent of the eligibles *not* enrolled even in equilibrium will have become eligible too recently to have yet joined the allowance program.

Both parts of the explanation contradict the notion that the eligible population is a fixed set of households. The eligible population is continuously turning over as individual households move into and out of eligibility. Newly eligible households do not enroll in the housing allowance program all at once. At any given time, a considerable fraction of eligible households will not yet have enrolled in the program.

In other words, two processes determine participation in the housing allowance program. The first is the movement of households into and out of eligibility. The second is the movement of eligible households into and out of the program. The participation rate at a given time is the ratio of households enrolled to those then eligible.

For a simple but powerful model of the participation rate, we assume that the first process is in equilibrium and then trace the second's approach to equilibrium. That is, we assume that the total

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*Herein, *participants* are households currently enrolled in the allowance program; they need not be receiving payments. Full participation in the program requires two steps: first, enrollment; and second, certification that the household's unit meets program standards. Allowance payments begin after completion of both steps. This note analyzes only the first step.

number of households and the proportion eligible are constant, even though particular households change eligibility status.

Of course, the number of eligible households may not be constant. However, modeling nonconstant eligibility greatly increases the complexity of the participation rate model while improving its explanatory power only a little.

Participation rates for nonelderly and elderly households differ both in level and in pattern, as a comparison of Tables 2 and 3 shows. At every year since the program began, the rate is higher for nonelderly households than for elderly households. At the end of two years the nonelderly rate has leveled off, but the elderly rate is still rising.

Because of those differences, we model the nonelderly and elderly participation rates separately (see Sec. II).* However, to build a general model that can be applied to both nonelderly and elderly households, we ignore the fact that some nonelderly households in one year are elderly households the next year. Extending the model to incorporate aging would improve its predictions, but at the cost of more complicated formulas.

Assuming equilibrium eligibility and ignoring household aging helps us highlight the basic dynamics of participation. Future models can relax both specifications and thereby achieve greater precision; here we choose simplicity.

Section III estimates the parameters for the nonelderly and elderly versions of the participation rate model. Section IV then aggregates the two models to explain the overall participation rate. The predicted rates for years 1, 2, and 3 of the housing allowance program agree remarkably well with the actual rates, indicating that our model comprehends the major causes of participation rates.

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*Our model has a general structure that could accommodate any number of subdivisions of the population. Here we distinguish nonelderly and elderly households, and find that the resulting model predicts overall participation rates very well. Further disaggregation might be useful, not so much for sharpening the predictions as for understanding the behavior of important subpopulations.

Table 2

PARTICIPATION IN THE HOUSING ALLOWANCE PROGRAM: NONELDERLY HOUSEHOLDS IN BROWN AND ST. JOSEPH COUNTIES

Years Since Program Began[a]	Number of Households			Participation Rate[b] (%)
	Noneligible	Nonenrolled Eligible	Enrolled Eligible	
Brown County				
0	31,900	4,600	0	0
1	31,900	2,660	1,940	42
2	31,900	2,420	2,180	47
3	31,900	2,550	2,050	45
St. Joseph County				
0	52,600	7,200	0	0
1	52,600	4,880	2,320	32
2	52,600	3,950	3,250	45
3	52,600	3,930	3,270	45
Both Counties				
0	84,500	11,800	0	0
1	84,500	7,540	4,260	36
2	84,500	6,370	5,430	46
3	84,500	6,480	5,320	45

SOURCE: Housing allowance office administrative records for years 1 to 3, and HASE surveys of households at baseline, Sites I and II.

NOTE: Total eligible and noneligible households are assumed to be constant, even though particular households in each category change.

[a]Calendar equivalents are as follows:

Years	Brown County	St. Joseph County
0	June 1974	December 1974
1	June 1975	December 1975
2	June 1976	December 1976
3	June 1977	December 1977

[b]Enrolled households as percent of eligible households, i.e., as percent of nonenrolled eligible households plus enrolled eligible households.

Table 3

PARTICIPATION IN THE HOUSING ALLOWANCE PROGRAM: ELDERLY HOUSEHOLDS IN BROWN AND ST. JOSEPH COUNTIES

Years Since Program Began[a]	Number of Households			Participation Rate[b] (%)
	Noneligible	Nonenrolled Eligible	Enrolled Eligible	
Brown County				
0	3,900	3,400	0	0
1	3,900	2,460	940	28
2	3,900	2,200	1,200	35
3	3,900	2,050	1,350	40
St. Joseph County				
0	6,100	8,400	0	0
1	6,100	7,450	950	11
2	6,100	6,350	2,050	24
3	6,100	5,540	2,860	34
Both Counties				
0	10,000	11,800	0	0
1	10,000	9,910	1,890	16
2	10,000	8,550	3,250	28
3	10,000	7,590	4,210	36

SOURCE: Housing allowance office administrative records for years 1 to 3, and HASE surveys of households at baseline, Sites I and II.

NOTE: Total eligible and noneligible households are assumed to be constant, even though particular households in each category change.

[a]Calendar equivalents are as follows:

Year	Brown County	St. Joseph County
0	June 1974	December 1974
1	June 1975	December 1975
2	June 1976	December 1976
3	June 1977	December 1977

[b]Enrolled households as percent of eligible households, i.e., as percent of nonenrolled eligible households plus enrolled eligible households.

II. MODELING THE PARTICIPATION RATE

Computing eligibility and participation rates requires the three overlapping counts of households given earlier in Table 1, which can be expressed as the vector

$$Y_t = \begin{pmatrix} H_t \\ E_t \\ P_t \end{pmatrix}, \tag{1}$$

where Y_t = household vector (overlapping states),
H_t = total households,
E_t = eligible households,
P_t = enrolled eligible households,
t = time.

Modeling eligibility and participation, on the other hand, requires mutually exclusive counts of households (as in Tables 2 and 3):

$$\mathcal{Z}_t = \begin{pmatrix} R_t \\ N_t \\ P_t \end{pmatrix}, \tag{2}$$

where $\mathcal{Z}_t$ = household vector (exclusive states),
R_t = noneligible households,
N_t = nonenrolled eligible households,
P_t = enrolled eligible households.

The two descriptions are connected by a sample transformation:

$$Y_t = F\,\mathcal{Z}_t\,, \tag{3}$$

where

$$F = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

The Z_t vector can change for three reasons: changes in the state of existing households, addition of new households (by household formation and inmigration), or subtraction of old households (by household dissolution and outmigration):

$$Z_{t+1} = M Z_t + A Z_t - S Z_t , \tag{4}$$

where M = matrix of transformation rates,

A = matrix of addition rates,

S = matrix of subtraction rates.

The transformation matrix is

$$M = \begin{pmatrix} 1-g & x & x \\ g & 1-x-n & p \\ 0 & n & 1-x-p \end{pmatrix} , \tag{5}$$

where g = rate of entrance to eligibility,

x = rate of exit from eligibility,

n = rate of enrollment in program,

p = rate of exit from program into eligibility.

Additions are assumed to be a constant fraction of noneligibles, nonenrolled eligibles, and enrolled eligibles. However, additions go directly into only the first two categories. To become enrolled, new households must be transformed by the M matrix. The addition matrix is

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & a \\ 0 & 0 & 0 \end{pmatrix} , \tag{6}$$

where a = rate of new household formation plus inmigration.

We assume the subtraction rates are the same for all categories of households. That gives us a diagonal subtraction matrix:

$$S = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix}, \tag{7}$$

where s = rate of old household dissolution plus outmigration.

To model the Y_t vector as a function of the flow rate parameters, we use Eqs. (3) and (4) and find that

$$Y_{t+1} = F(M + A - S)Z_t , \tag{8}$$

then use Eq. (3) again to yield

$$Y_{t+1} = F(M + A - S)F^{-1} Y_t . \tag{9}$$

The required inverse of the F matrix is

$$F^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} . \tag{10}$$

The indicated matrix operations yield

$$F(M + A - S)F^{-1} = \begin{pmatrix} 1+a-s & 0 & 0 \\ g & 1+a-s-x-g & 0 \\ 0 & n & 1-n-s-x-p \end{pmatrix} . \tag{11}$$

Substituting Eq. (11) into Eq. (9) gives the difference equations describing changes in household counts:

$$H_{t+1} - H_t = (a - s)H_t , \tag{12}$$

$$E_{t+1} - E_t = g\ V_t + (a - s - x - g)E_t , \tag{13}$$

$$P_{t+1} - P_t = n\ E_t - (n + s + x + p)P_t . \tag{14}$$

Under the assumption that the number of households is constant, i.e., $H_t = H$, Eq. (12) implies that the addition rate equals the subtraction rate:

$$a = s \ . \tag{15}$$

Under the assumption that the number of eligible households is constant, i.e., $E_t = E$, Eq. (13) implies the following eligibility rate:

$$\frac{E}{H} = \frac{g}{g + x} \ . \tag{16}$$

Those assumptions allow us to transform Eq. (14) into the differential equation

$$\frac{dr(t)}{dt} = n - [n + s + x + p]\, r(t) \ , \tag{17}$$

where $r(t) = P(t)/E$ = the participation rate. Solving Eq. (17) yields our model of the participation rate:

$$r(t) = \left[\frac{n}{n + s + x + p}\right]\left[1 - e^{-(n+s+x+p)t}\right] \ . \tag{18}$$

Two general conclusions flow from Eq. (18). The first is that the equilibrium participation rate equals the enrollment rate, n, divided by the sum of the enrollment rate and the termination rate, $s + x + p$.*

$$\lim_{t \to \infty} r(t) = \frac{n}{n + s + x + p} \ . \tag{19}$$

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*It was not necessary to solve the differential equation to achieve this result. An equivalent derivation defines equilibrium by setting the left-hand side of Eq. (14) to zero, then solves for $P_t/E = n/(n + s + x + p)$.

The three components of the termination rate are the rates at which households leave the site, s, leave eligibility, x, and leave the program while remaining eligible, p.

That the equilibrium participation rate increases with the enrollment rate and decreases with the exit rate makes sense: The faster the inflow and the slower the outflow, the higher the number of households in the program ought to be.

The second conclusion is that the length of time the participation rate takes to reach 95 percent of the equilibrium participation rate is inversely proportional to the sum of the enrollment and termination rates:

$$r(t) = .95\left[\frac{n}{n + s + x + p}\right] \quad \text{if} \quad t = \frac{3.00}{n + s + x + p} . \tag{20}$$

That a higher enrollment rate causes a more rapid approach to equilibrium is a plausible finding, but why should a higher termination rate have the same effect? The reason is that a higher termination rate lowers the equilibrium participation rate, making it easier to attain.

III. ESTIMATING PARAMETERS

The derivation of Eq. (18) shows that the rate of participation in the housing allowance program depends on only the enrollment rate and the termination rate. Tables 4 and 5 present the empirical evidence on those rates.

During the first three years of the housing allowance program, the annual enrollment rate for nonelderly households averaged 51 percent, and showed strong (and opposite) trends in both sites (see Table 4).* The annual termination rate for such households averaged 45 percent, and while roughly the same in both sites, it was considerably lower in the first year than in the second or third years. Presumably administrative delays in the program's first year caused some terminations to be counted in the second year.

The comparable figures for elderly households are considerably lower (see Table 5). The average annual enrollment rate was 20 percent, the average annual termination rate, 21 percent. The enrollment rates again show opposite trends in the two sites.

Our model distinguishes three components of the termination rate: the exit rate from eligibility, x, due to escape from poverty or other reasons; the exit rate from the program into eligibility, p, and the subtraction rate, s, due to outmigration or death. Table 6 classifies reported reasons for termination according to that scheme. The first entry ("failed to recertify") is troublesome because we can only infer those so classified are no longer eligible. The remaining entries reflect the enrollee's explanation or the housing allowance office's decision.

Table 7 uses the data in Table 6 to decompose the overall termination rates reported in Tables 4 and 5. The table shows that the

*Those trends are consistent with a hypothesis that the number of eligible households is decreasing in Brown County and increasing in St. Joseph County. However, the present analysis reveals the major causes of participation rates without introducing the complexity of varying eligibility and enrollment rates.

Table 4

ENROLLMENT AND TERMINATION RATES IN THE HOUSING ALLOWANCE PROGRAM: NONELDERLY HOUSEHOLDS IN BROWN AND ST. JOSEPH COUNTIES

Time	Annual Enrollment		Annual Terminations[a]	
	Number of Households	Rate[b] (%)	Number of Households	Rate[c] (%)
Brown County				
Year 1	2,100	58	160	16
Year 2	1,350	53	1,110	54
Year 3	1,060	43	1,190	56
St. Joseph County				
Year 1	2,580	43	260	22
Year 2	2,320	53	1,390	50
Year 3	2,210	56	2,190	67
Both Counties				
Year 1	4,680	48	420	20
Year 2	3,670	53	2,500	52
Year 3	3,270	51	3,380	63

SOURCE: Housing allowance office administrative records for years 1 to 3, Sites I and II.

[a]Terminations estimated as a residual to make enrolled at start of year plus enrollment during year less terminations during year equal enrolled at end of year.

[b]Annual enrollment as fraction of nonenrolled eligibles at mid-year (estimate by averaging counts at start and end of year).

[c]Annual terminations as fraction of enrolled at mid-year (estimated by averaging counts at start and end of year).

Table 5

ENROLLMENT AND TERMINATION RATES IN THE HOUSING ALLOWANCE PROGRAM: ELDERLY HOUSEHOLDS IN BROWN AND ST. JOSEPH COUNTIES

Time	Annual Enrollment		Annual Terminations[a]	
	Number of Households	Rate[b] (%)	Number of Households	Rate[c] (%)
Brown County				
Year 1	1,070	37	130	28
Year 2	500	21	240	22
Year 3	400	19	250	20
St. Joseph County				
Year 1	1,060	13	110	23
Year 2	1,330	19	230	15
Year 3	1,350	23	540	22
Both Counties				
Year 1	2,130	20	240	25
Year 2	1,830	20	470	18
Year 3	1,750	22	790	21

SOURCE: Housing allowance office administrative records for years 1 to 3, Sites I and II.

[a]Terminations estimated as a residual to make enrolled at start of year plus enrollment during year less terminations during year equal enrolled at end of year.

[b]Annual enrollment as fraction of nonenrolled eligibles at mid-year (estimated by averaging counts at start and end of year).

[c]Annual terminations as fraction of enrolled at mid-year (estimated by averaging counts at start and end of year).

Table 6

REASONS FOR TERMINATION OF ENROLLMENT IN THE HOUSING ALLOWANCE PROGRAM: FIRST TWO PROGRAM YEARS IN BROWN AND ST. JOSEPH COUNTIES

Reason for Termination	Percentage Distribution of Terminations	
	Nonelderly Households	Elderly Households
Escape from Poverty		
Failed to recertify[a]	49.3	30.9
Income too high	22.7	10.1
Assets too high	0.5	2.7
Fraud (found ineligible)	0.6	0.1
Could not identify reason	0.2	1.1
All other reasons	3.7	5.3
Total	77.0	50.2
Other Exits from Eligibility		
Change in household composition	1.9	0.4
Moved to subsidized housing	1.8	11.0
Joined other assistance program	0.1	0.7
Moved to nursing home	0.0	5.6
Total	3.8	17.7
Exit from Program to Eligibility		
Allowance too small	4.0	5.7
Feels assistance not needed	2.2	3.0
Failed housing evaluation, no move	2.2	4.6
No lease, no move	0.5	1.9
Spent too little on housing expense	1.0	0.7
Failed to allow housing evaluation	0.2	0.0
Administrative burden	0.3	1.2
Confidentiality	0.2	0.3
Welfare image	0.0	0.4
Total	10.6	17.8
Subtraction of Household		
Outmigration from county	7.9	4.5
Death	0.7	9.8
Total	8.6	14.3
Grand total	100.0	100.0

SOURCE: Housing allowance office administrative records for years 1 and 2, Sites I and II.

[a]Did not respond to repeated recertifiction notices, so enrollment was terminated.

Table 7

COMPONENTS OF THE RATE AT WHICH ENROLLED HOUSEHOLDS TERMINATE ENROLLMENT IN THE HOUSING ALLOWANCE PROGRAM

Reason for Termination	Percent of Terminations		Components of Annual Termination Rate (%)	
	Nonelderly Households	Elderly Households	Nonelderly Households	Elderly Households
Exit from eligibility:				
Escape from poverty	77.0	50.2	34	10
Other	3.8	17.7	2	4
Exit from program to eligibility	10.6	17.8	5	4
Subtraction of household	8.6	14.3	4	3
Total	100.0	100.0	45	21

SOURCE: Tables 4, 5, and 6.

NOTE: Estimated total termination rates are averages of the annual termination rates during the first three years of the allowance program, given in Tables 4 and 5.

termination rate for nonelderly households is higher than for elderly households primarily because nonelderly households escape poverty more readily. The annual rate of escape from poverty is 34 percent for nonelderly households, vs. only 10 percent for elderly households.

Our estimate of the rate at which nonelderly households escape poverty compares favorably with the 31 percent estimate found in Frank Levy's analysis of the University of Michigan Panel Study on Income Dynamics.* Levy studied individuals not households, and he used the Social Security Administration poverty standard (the "Orshansky Standard") not housing allowance eligibility rules. Nevertheless, his estimate is the best currently available.

Table 8 brings together our estimates of all the parameters defined in Sec. II. The addition rate, a, and the entrance rate to eligibility, g, are estimated for completeness. Our model of the participation rate requires only the enrollment rate, n, and the total termination rate, $s + x + p$.

How Big Is the American Underclass, Graduate School of Public Policy, University of California, Berkeley, June 1976, p. 25.

Table 8

POPULATION, ELIGIBILITY, AND PARTICIPATION CHANGES FOR HOUSEHOLDS IN BROWN AND ST. JOSEPH COUNTIES

Parameter		Annual Rate (%)	
Symbol	Description	Nonelderly Households	Elderly Households
	Population Changes		
a	Addition of households[a]	4	3
s	Subtraction of households[b]	4	3
	Eligibility Changes		
g	Entrance to eligibility[c]	5	17
x	Exit from eligibility[d]	36	14
	Participation Changes		
n	Enrollment in program[e]	51	20
p	Exit from program to eligibility[f]	5	4

SOURCE: Tables 4, 5, 6, and 7.

[a]Formation of new households plus inmigration; rate estimated assuming population is constant.

[b]Dissolution of old households plus outmigration.

[c]Estimated with Eq. (16), assuming number of eligibles is constant.

[d]Escape from poverty plus other exits from eligibility (see Table 7).

[e]Estimated by the average of the annual enrollment rates during the first three years of the allowance program (see Tables 4 and 5).

[f]See Table 7.

IV. USING THE PARTICIPATION MODEL

Substituting the parameter estimates of Table 8 into Eq. (18) gives explicit models of the participation rate for nonelderly households:

$$r(t) = .531\left(1 - e^{-.96t}\right), \tag{21}$$

and for elderly households:

$$r(t) = .488\left(1 - e^{-.41t}\right), \tag{22}$$

where $r(t)$ = fraction of eligible households enrolled in the housing allowance program,

t = time (in years).

Averaging the two models yields the participation model for all households. (The unweighted average is correct because there are equal numbers of nonelderly and elderly eligibles in our sites; refer to Table 1.)

$$r(t) = .266\left(1 - e^{-.96t}\right) + .244\left(1 - e^{-.41t}\right). \tag{23}$$

Table 9 predicts participation rates for the end of each year of the experimental housing allowance program. Table 10 shows that predicted and actual rates are remarkably close, especially when nonelderly and elderly rates are averaged into the overall participation rates. The largest prediction errors occur in year 3 and are overestimates of nonelderly participation and underestimates of elderly participation, partly because the model does not recognize that some households who are nonelderly at the start of the program become elderly by the third year.

To find how long newly eligible households take to enroll in the housing allowance program, we alter the participation rate model to a cohort tracking version. Noneligibility becomes a trapping state--i.e,

Table 9

PARTICIPATION RATES BY TIME SINCE PROGRAM BEGAN, PREDICTED FROM PARTICIPATION MODEL

Years Since Program Began	Percent of Eligibles Participating		
	Nonelderly Households	Elderly Households	All Households
0	0	0	0
1	32.8	16.4	24.6
2	45.3	27.3	36.3
3	50.1	34.5	42.3
4	52.0	39.3	45.7
5	52.7	42.5	47.6
6	52.9	44.6	48.8
7	53.0	46.0	49.5
8	53.1	47.0	50.1
9	53.1	47.6	50.4
10	53.1	48.0	50.6

SOURCE: Calculated from Eqs. (21), (22), and (23).

NOTE: Parameters of participation model are estimated from combined data for Brown and St. Joseph counties, years 1 to 3.

when portions of an eligible cohort once cease to be eligible, they are never allowed to return to the cohort--by setting the eligibility entrance rate, g, to zero. New entrants are not allowed into an eligible cohort by setting the addition rate, a, to zero.

Those changes to the model presented in Sec. II transform Eqs. (13) and (14) into difference equations that translate into the following differential equations:

$$\frac{dE(t)}{dt} = -(s + x)\, E(t) , \tag{24}$$

$$\frac{dP(t)}{dt} = n\, E(t) - (n + s + x + p)\, P(t) . \tag{25}$$

The solutions to Eqs. (24) and (25) are

Table 10

PREDICTED VS. ACTUAL PARTICIPATION RATES: COMBINED DATA FOR BROWN AND ST. JOSEPH COUNTIES

Years Since Program Began	Percent of Eligibles Participating		
	Predicted	Actual	Error (predicted - actual)
Nonelderly Households			
1	33	36	-3
2	45	46	-1
3	50	45	+5
Elderly Households			
1	16	16	0
2	27	28	-1
3	34	36	-2
All Households			
1	25	26	-1
2	36	37	-1
3	42	40	+2

SOURCE: Tables 2, 3, and 9.

$$E(t) = E(0)\ e^{-(s+x)t}\ , \tag{26}$$

and

$$P(t) = \left[\frac{n\ E(0)}{n + p}\right]\left[e^{-(s+x)t} - e^{-(n+s+x+p)t}\right]\ . \tag{27}$$

Dividing participants in the allowance program, $P(t)$, by the total number of households in the cohort that remain eligible, $E(t)$, gives the rate of participation in the program as a function of the time since eligibility began:

$$\frac{P(t)}{E(t)} = \left[\frac{n}{n + p}\right]\left[1 - e^{-[n+p]t}\right]\ , \tag{28}$$

where $P(t)/E(t)$ = cohort's participation rate,

t = time since eligibility began.

The results of calculations using Eq. (28) are given in Table 11. For nonelderly households, participation rates rise relatively rapidly with time since eligibility began. Only 22 percent of the nonelderly households that have been eligible for half a year are participants, as opposed to 82 percent of those that have been eligible for four years. Elderly households enroll more slowly, however, and even after four years only 51 percent of those remaining eligible are enrolled in the program.

Table 11

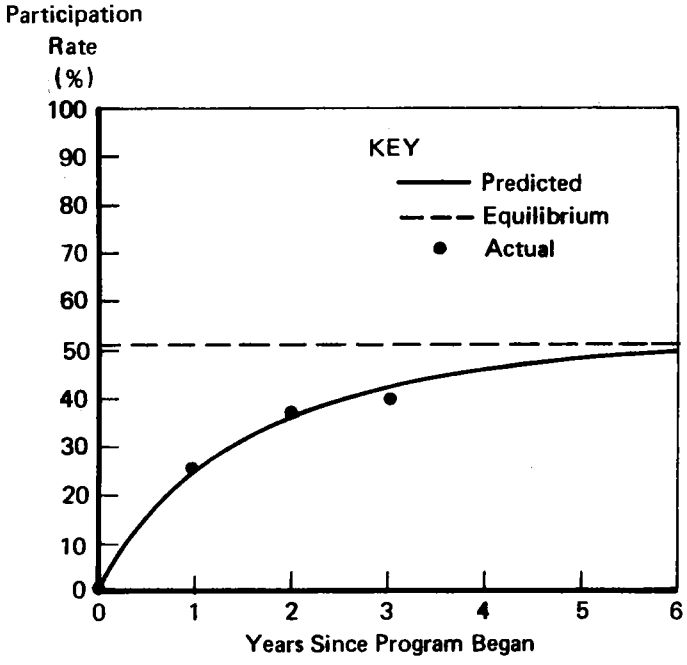
PARTICIPATION RATES FOR NONELDERLY AND ELDERLY HOUSEHOLDS BY TIME SINCE ELIGIBILITY BEGAN

Years Since Eligibility Began	Participation Rate (%)	
	Nonelderly Households	Elderly Households
0	0.0	0.0
0.5	22.2	9.4
1	39.2	17.8
2	61.8	31.8
3	74.9	42.8
4	82.4	51.4

SOURCE: Equation (28) and Table 8.

V. CONCLUSIONS

Three years after the housing allowance program began in Brown and St. Joseph counties, the participation rate for eligible households was only four-fifths the 51 percent equilibrium rate predicted by the present analysis. Not until the program has operated 5.5 years will the participation rate reach 95 percent of the equilibrium rate, according to model predictions (see the figure below). The interaction of enrollment and termination rates causes the participation rate to be so far below 100 percent. Since the equilibrium participation rate equals the enrollment rate (fraction of nonenrolled eligibles that join the program in a year) divided by the sum of the enrollment rate and the termination rate (fraction of enrollees that leave the program that year), the equilibrium participation rate would be 100



SOURCE: Tables 9 and 10.

Figure— Predicted versus actual participation in the housing allowance program

percent only if the termination rate were zero--that is, only if there were a permanent group of participants.

The 51 percent equilibrium participation rate may strike some observers as surprisingly low. But it is not low compared with other government transfer programs. The rates for the housing allowance program in Brown and St. Joseph counties and the welfare program in New York City, for example, are remarkably similar.

Table 12 presents the enrollment and termination rates for the housing allowance program, and compares actual participation rates in the program's third year with predicted equilibrium participation rates. The 40 percent overall actual participation rate is below the 51 percent predicted equilibrium rate because enrollment had not reached equilibrium.

Table 12

DYNAMICS OF PARTICIPATION IN THE HOUSING ALLOWANCE PROGRAM: BROWN AND ST. JOSEPH COUNTIES, 1977

Type of Household	Annual Flow Rates (%)		Participation Rates (%)	
	Enrollment[a]	Termination[b]	Actual[c]	Equilibrium[d]
Nonelderly	51	45	45	53
Elderly	20	21	36	49
All	35	32	40	51

SOURCE: Tables 4, 5, 9, and 10.

[a]Fraction of nonenrolled eligible households that enroll in the program per year.

[b]Fraction of enrolled households that leave the program per year.

[c]Ratio of enrolled households to all eligible households at the end of the program's third year (June 1977 in Brown County and December 1977 in St. Joseph County).

[d]Equilibrium that would occur if the flow rates and the number of eligible households remained constant.

Table 13 presents comparable flow and participation rates for the welfare caseload in New York City. Again, the overall actual participation rate (52 percent) is below the predicted equilibrium participation rate (56 percent), and for the same reason--the New York City welfare caseload was not in equilibrium in March 1970; it was growing.

Table 13

DYNAMICS OF PARTICIPATION IN THE WELFARE PROGRAM: NEW YORK CITY, 1970

Type of Household	Annual Flow Rates (%)		Participation Rates (%)	
	Enrollment[a]	Termination[b]	Actual[c]	Equilibrium[d]
Single-parent[e]	248	19	88	93
Elderly[f]	11	23	28	32
Other[g]	58	66	43	47
All	49	35	52	56

SOURCE: C. Peter Rydell, Thelma Palmerio, Gerard Blais, and Dan Brown, *Welfare Caseload Dynamics in New York City*, The New York City Rand Institute, R-1441-NYC, October 1974, pp. 36-40.

[a]Fraction of eligible nonrecipient cases that join the welfare rolls per month multiplied by 12 to give the equivalent annual rate.

[b]Fraction of welfare cases that close each month, multiplied by 12 to give the equivalent annual rate.

[c]Ratio of welfare caseload to all eligible cases, 31 March 1970.

[d]Equilibrium that would occur if the flow rates and the number of eligible cases remained constant.

[e]Aid to Families with Dependent Children.

[f]Old-age assistance.

[g]Aid to Blind, Aid to Disabled, Aid to Families with Unemployed Parent, and Home Relief.

Especially in the welfare examples, equilibrium participation rates vary (from 32 to 93 percent) by type of household. The occurrence of high participation rates naturally raises the issue whether low participation rates can be increased.

One method of increasing participation rates would be to increase enrollment rates. To see the large increase potentially possible, compare elderly households in the housing allowance program with single-parent welfare cases. The two groups have about the same termination rate (21 vs. 19 percent a year), but the enrollment rate of the elderly households is less than a tenth that of single-parent welfare eligibles (20 vs. 248 percent a year).* If the elderly households had the single-parent welfare enrollment rate, their equilibrium participation rate would be 92 percent instead of 49 percent.

Another method of increasing participation rates would be to lower the termination rate. Most termination is escape from poverty, and no policymaker would recommend decreasing that component. The only other part of the termination rate that could be controlled by allowance program parameters is the termination of households that remain eligible (5 percent annually for nonelderly households and 4 percent annually for elderly households). But even if those rates could be reduced to zero, the overall equilibrium participation rate would only increase from 51 to 55 percent.

This discussion suggests that the only potential for large increases in participation rates lies in raising enrollment rates, the basis of which--individual enrollment decisions--is not well understood. One fruitful line of research on the determinants of enrollment rates might be to model enrollment with a benefit-cost framework: If the expected present value of benefits minus enrollment costs exceeds zero, a household will enroll. Whereas that view appears incongruent with the stochastic model of enrollment presented here, it is actually compatible.

The benefit-cost model, while deterministic in theory, is stochastic in practice, in that households' expected net benefit is replaced by actual net benefit, imperfectly measured. Assuming that

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*As the example shows, enrollment rates are not necessarily less than 100 percent. Theoretically, there is no upper limit to the rate at which enrollment can occur. Of course, the eligible population must be continuously replenished for an enrollment rate to be sustained.

measured net benefit is positively correlated with expected net benefit, the probability that a household will enroll in the housing allowance program is positively related to its measured net benefit. But a measured net benefit greater than zero does not assure enrollment because of the imperfect link to expected net benefit; thus the operational model is stochastic.

A stochastic benefit-cost model of enrollment would be a logical extension of this note's analysis. We have explained how flow rates--enrollment and termination--interact to determine participation rates; the next step is to understand the flow rates.