Markov Chain Model of Rent Burden in the Housing Choice Voucher Program

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Abstract
This study models rent burden in the Housing Choice Voucher Program as a Markov Chain. The model predicts rent burden with program tenure, using longitudinal household data for 2000 through 2009. Results indicate rent burden increases for many years after admission. Consistent with results for unassisted low-income families, the model also predicts considerable mobility across burden categories over time. The rent burden formula indicates that HUD policy and housing agency policy should not be considered in isolation. Estimates imply that their interaction has an effect large in both magnitude and statistical significance. A limitation of the Markov Chain model is that it doesn't estimate variance; I demonstrate a simple method for doing so via bootstrapping.

Acknowledgements
I thank Chun-Hung Chen, Mark Shroder, and Scott Susin for helpful comments, along with participants at a HUD workshop in April 2010. I also thank Mark Perdue for valuable assistance constructing the longitudinal data files.

I Introduction

Since 1937, the United States has made access to affordable housing a national priority. For a numerous reasons, assistance has been provided through housing subsidies rather than general income transfers.

Affordable housing assistance was initially provided through public housing projects run by local Public Housing Agencies (PHAs). In 1965, PHAs began experimenting with providing opportunities in privately owned rental units.[1] Private opportunities have been provided through a variety of programs.

There are many arguments for providing assistance in privately owned buildings instead of public housing. The primary motivation for increasing private sector housing choices has been expanding social and economic opportunities for low-income households receiving housing assistance. Another argument is that private owners might have better incentives for operational efficiency, thus lowering program costs.

Since 1995 the largest private options have been Certificates and the Housing Choice Voucher Program (HCVP). In 1995 there were 1.2 million households in public housing, compared to 1.3 million certificates and vouchers.[2]

Certificates have been phased out over time, while Housing Choice Vouchers have increased dramatically. Today, HUD provides assistance to about 1.1 million households in public housing projects, and about 2 million through the Housing Choice Voucher Program. HCVP is the now largest U.S. rental assistance program, with an annual budget over $16 billion.

Voucher recipients choose their own rental units, and rent is paid for partially by the household and partially through HUD payments to landlords. The voucher program is administered by approximately 2600 local Public Housing Agencies (PHAs). Among the PHAs' responsibilities are selecting applicants to receive vouchers, annual verification of household income, and determination of each household's rent responsibility.

The Housing Assistance Payment (HAP) under the program is the difference between 1) the minimum of gross rent (rent plus utility costs) and the Payment Standard, and 2) the Total Tenant Payment (TTP). Payment Standards and TTP are both set by PHAs. TTP is the minimum household contribution, which is usually about 30 % of household adjusted income.

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[1] See http://www.mphaonline.org/section8.cfm for a concise history of HUD rental assistance programs.

[2] http://www.huduser.org/portal/datasets/assthsg/statedata96/index.htm

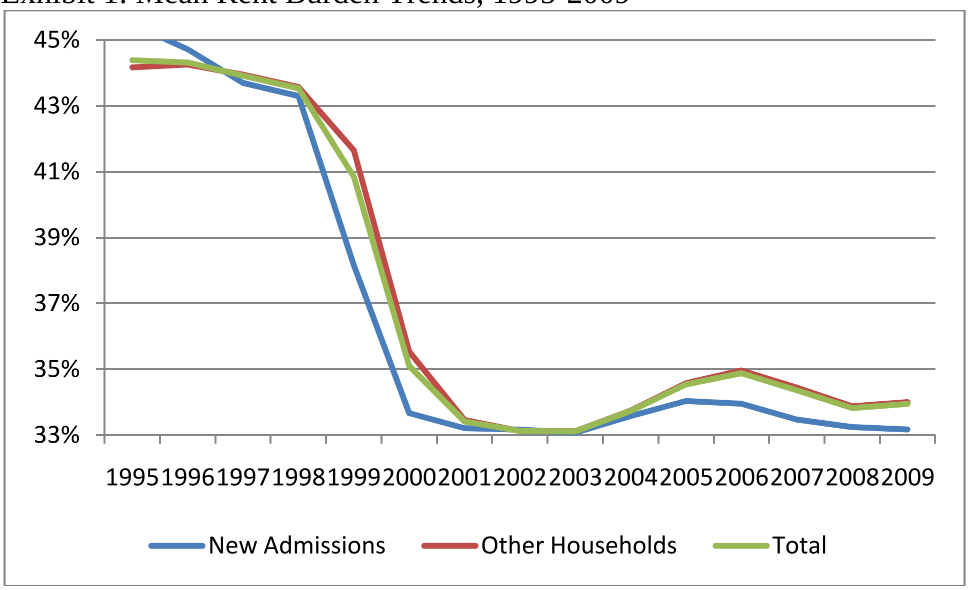
If the Payment Standard set by the PHA is in-line with actual rents, households should be able to find units meeting HUD quality standards at burden levels around 30 %. Yet in reality, many households face burdens considerably greater than 30 %.

Starting in 1998, when admitted to the program or moving into a new unit, a family's rent burden is required to be between 30 and 40 % of adjusted household income. In other years, rent burden has a 30 % floor with no ceiling.

Data indicate the ceiling for admissions and move-ins is not strictly enforced by all PHAs. In 2009, 4.216 % of new admissions had rent burdens of at least 41 %, as did 5.480 % of movers.

Exhibit 1 reports trends in rent burden for new admissions, other households, and total households from 1995-2009. After restrictions were placed on burdens at admission or move-in in 1998, average rent burdens fell dramatically. Around 2003 burdens increased, and fell again starting around 2006. As of 2009, the mean burden was 33.950 % overall, 33.172 % for new admissions, and 34.003 % for other households.

Exhibit 1: Mean Rent Burden Trends, 1995-2009

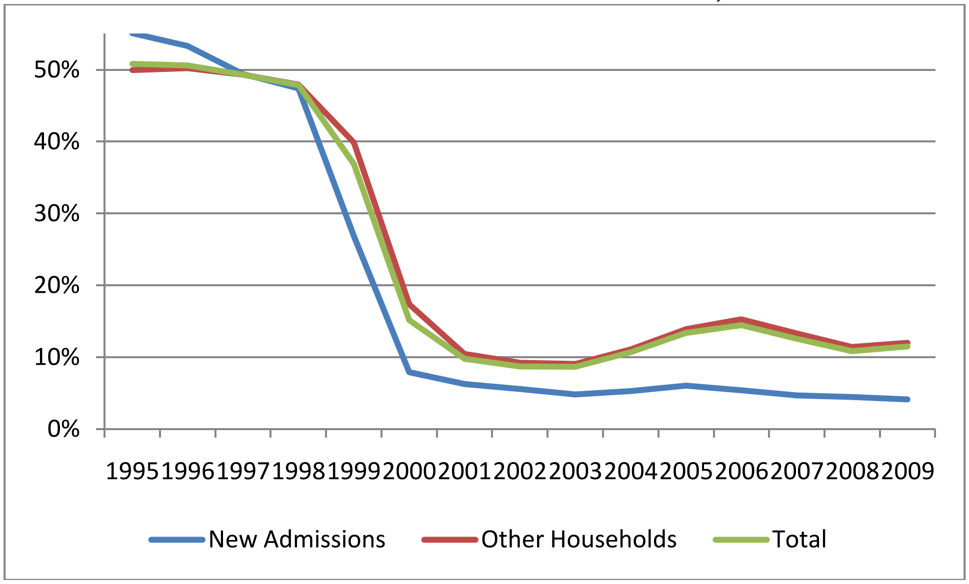


N=15,913,767. Source: author's calculations using MTCS/PIC data 1995-2009.

While mean burdens in recent years may seem reasonable, many households have burdens considerably higher than the mean. Exhibit 2 reports trends for percentages of households with burdens of at least 41 %. Since 2000, the fraction of households with burdens above 40 % has fluctuated. In 2009, 11.502 % of households fell in this category.

Rent burdens above 30 % don't necessarily imply inadequate subsidies. Indeed, the program is designed to foster choice. Households may choose burden levels above 30 % for a variety of reasons. For instance, households may have a preference for larger homes, newer homes with better amenities, or homes in better neighborhoods.

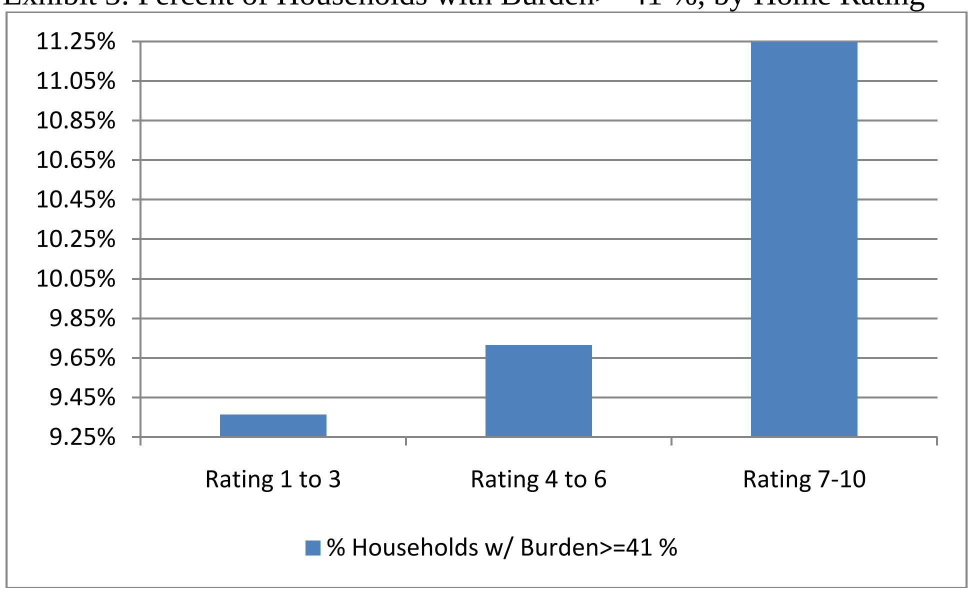
Exhibit 2: Percent of Households with Burdens >=41 %, 1995-2009



N=15,913,767. Source: author's calculations using MTCS/PIC data 1995-2009.

Between 2000 and 2002, HUD conducted a survey of HCVP households.[3] Focusing on housing quality, one question asked residents to rate their home on a scale of 1-10. Exhibit 3 reports percentages of respondents with burdens of at least 41 % for three categories of home ratings: 1-3, 4-6, and 7-10.

Exhibit 3: Percent of Households with Burden>=41 %, by Home Rating



N=310,314. Source: CSS and MTCS data, 2000-2002

High burdens are more prevalent for households with higher household ratings. For homes with ratings of 1-3, 9.362 % have burdens of at least 41 %. Among families rating their home 7 or above, 11.246 % have high burdens.

[3] Dubbed the Customer Satisfaction Survey (CSS), see Mast (2009) for more information.

Policy implications depend not only on the number of households with high rent burdens at any given time, but also on the tendency of households to have persistent periods of high burden. While prior research suggests considerable turnover in the population of households with high rent burdens (Martin et al. 2005), prolonged burden may disproportionately affect the most disadvantaged households (Susin 2007).

This study models rent burden in the HCVP as a Markov Chain, using longitudinal data on households admitted to the program between 2000 and 2009. The model is a simple and powerful tool for examining rent burden patterns with time in program. Estimates imply increasing burden with program tenure, and considerable household mobility across rent burden categories. The model also estimates large differences in burden due to HUD and PHA policies.

One limitation of Markov Chain models is that they don't estimate variance. I demonstrate a simple method for doing so via bootstrapping.

The Markov Chain model predicts burden for households remaining in the program past their admission year. Large variation in exit rates with program tenure make exit unsuitable for Markov Analysis. Instead, I examine the relationship between burden and program exit with survival analysis. Estimates indicate low chances of exit in the admission year. In subsequent years, exit rates are larger with moderately burdened households most likely to exit. Households admitted with burdens below 32 % are estimated to stay in the HCVP program about .4 years longer than other families.

The next section summarizes the relevant literature. Program guidelines for determining rent burden are then discussed, followed by a description of the data. The relationship between burden and program tenure is then considered, followed by Markov Chain analysis. Conclusions are summarized in the final section.

II Literature Review

Rent Burden Studies Many studies have examined rent burden in the general population (i.e., Hill 2003, HUD 1998, 2000, 2001, 2003). Martin et al. (2005) analyze data from the American Housing Survey (AHS) and 2 years of longitudinal data from the Survey of Income of Program Participation (SIPP). They find much churn in the population with high rent burdens.

Susin (2007) analyzes duration of rent burden for low-income families, using 3 years of SIPP data. The focus of his study was identifying household characteristics related to prolonged periods of high burden. He reports numerous indicators of need, such as non-employment and receipt of Supplemental Security Income (SSI), are associated with persistent high burden.

HUD (2007) compare rent burden estimates from the AHS with SIPP estimates, which were mostly adopted from Susin (2007). Compared to the SIPP, the AHS has much better utility cost data. Susin (2007) and HUD (2007) use regional AHS data to impute utility costs for SIPP participants.

A few studies have examined rent burden in the HCVP program. McClure (2005) is the most recent, using the same data source as this study (HUD's MTCS data system, described below).

Analyzing household data for 2000 to 2002, he studies how rent burden varies with a large number of household, housing market, and policy variables. McClure finds high rent burdens are most prevalent among very low-income families (relative to other low-income HCVP households).

Using only three years of data, McClure's study doesn't indicate whether periods of extremely low income (and high rent burden) are "chronic or transitory" (McClure 2007, p. 18). This study attempts to extend the literature by analyzing rent burden patterns over a 10 year period (2000-2009).

Markov Chain Models According to Ross (2007), A Markov Chain model is a discrete time stochastic process where the conditional distribution of any future state is independent of past states, depending only on the current state. Processes that satisfy this assumption are referred to as stationary, because the probability of transitioning to another state is fixed over time. Such processes are also referred to as memoryless, because the past has no predictive power.

HCVP rent contracts are effective for a discrete time period (typically 1 year). And while dynamics of HCVP rent burden have yet to be examined, past research on unassisted low-income households indicates considerable rent burden churn (Martin et al. 2005, Susin 2007, HUD 2007). Thus HCVP rent burden may qualify as a discrete time stochastic process. And while rent burden in many prior years may influence future burden, the most recent is likely the most important. Thus a Markov Chain may be a reasonable model of HCVP rent burden.

Susin (2007) uses logistic regression to predict rent burden duration. McClure (2005) only reports summary statistics. To my knowledge, this is the first study modeling rent burden as a Markov Chain. Many researchers have applied Markov Chain models to other housing market topics, however. An early example is Clark (1965), who uses a Markov Chain to predict geographic movement of households.

A Markov Chain model is a simple and convenient method to analyze patterns of rent burden over time. One limitation of the Markov Chain model is that rent burden must be categorized. Yet it is really a category of rent burden that is of policy concern: those above some threshold level (traditionally between 40 % and 50 %) considered to cause economic hardship on families.

The most critical limitation of the Markov Chain model is the stationarity assumption. The definition of states in Markov Chains is flexible, however. And Ross (2007) demonstrates that a more generalized stochastic process can be transformed into a Markov Chain by defining states based on multiple time periods. I follow this strategy by defining rent burden categories based on two years of data.

In general, Markov Chain models are used more for predicting than explaining changes in a variable. Regression is more practical for estimating marginal effects for large numbers of explanatory variables. Yet Markov Chain states can be defined in ways useful for policy analysis. This study demonstrates how a Markov Chain model can predict the impact of HUD policy (Fair Market Rent) and PHA policy (the Payment Standard) on rent burden.

While regression may be a better model for exploring causality, many variables correlated with rent burden are choice variables potentially influenced by rent burden. For instance, while rent burden varies with neighborhood choice, rent burden may also influence location decisions. Thus many variables related to rent burden (i.e., neighborhood, unit size, housing quality, moves) are potentially endogenous with respect to rent burden. Income as an explanatory variable can also be problematic in regression analysis if measured with error, because income is also in the denominator of rent burden.

McClure (2005) thoroughly examined HCVP Rent burden variation across demographic groups. While I do examine differences across broad household categories, I do not make estimates for particular racial, gender, or geographic subgroups. However the Markov Chain states in this study could be defined in ways to make estimates for narrower demographic categories.

III Rent Burden Formula

Rent burden (burden) equals the difference between gross rent including utility payments (rent) and the Housing Assistance Payment (HAP), divided by household adjusted income (income):

$$\text{burden} = [\text{rent} - \text{HAP}]/\text{income} \qquad \text{(eq1)}$$

HAP is the difference between 1) the minimum of gross rent and the Payment Standard (PS), and 2) the Total Tenant Payment (TTP):

$$\text{HAP} = \min(\text{rent}, \text{PS}) - \text{TTP} \qquad \text{(eq2)}$$

TTP is the minimum household contribution. It is calculated as the maximum of four values: 1) 30 % of household adjusted income, 2) 10 % of gross income, 3) the welfare rent, and 4) the PHA minimum rent. For the vast majority of households, TTP is close to 30 % of household adjusted income. Under this assumption, eq2 can be rewritten as:

$$\text{HAP} = \min(\text{rent}, \text{PS}) - .3*\text{income} \qquad \text{(eq3)}$$

Note that until 1998, households could keep the difference between Payment Standards and rent when rent fell below Payment Standards. Thus households had an incentive to search for units renting below the Payment Standard in order to reduce their rent burdens. Now, however, the minimum rent burden a household can incur is at least 30 %. Thus households have no incentive to search for qualifying units renting below the Payment Standard in order to reduce their rent burden.

Yet households may select units with gross rent below the payment standard for a variety of other reasons. Indeed, a slight majority of households fall in this category. Accordingly, let H1 equal the maximum of zero and Payment Standard minus gross rent:

$$\text{H1} = \max(0, \text{PS} - \text{rent}) \qquad \text{(eq4)}$$

Thus eq3 can be rewritten as follows, where H1 is nonnegative:

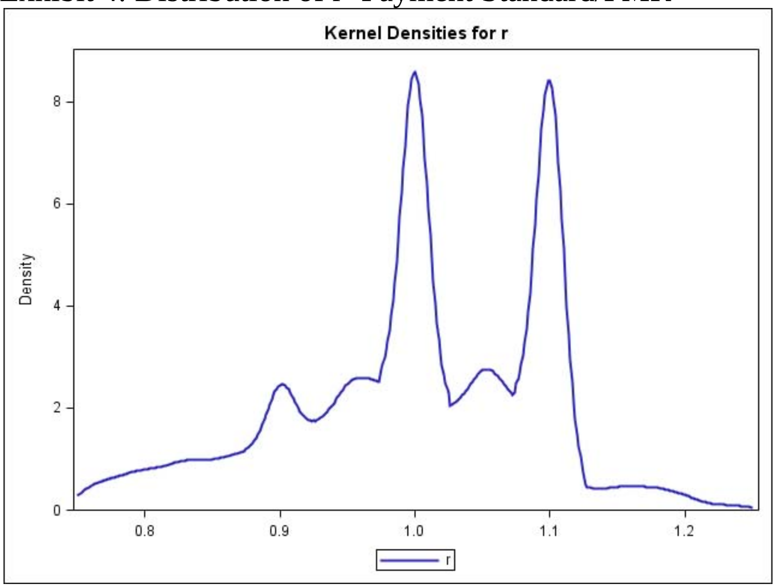
$$\text{HAP} = \text{PS} - \text{H1} - .3*\text{income} \qquad \text{(eq5)}$$

HUD attempts to set Fair Market Rent (FMR) at 40 % of the distribution for a unit with a given number of bedrooms in each FMR region. In tight markets, FMR is sometimes set at 50 % of rent distribution. Let r equal the ratio of the Payment Standard to FMR:

$$r = \text{Payment Standard}/\text{FMR} \qquad \text{(eq6)}$$

Exhibit 4 depicts the distribution of r. It has a mean of 1.003, a median of 1, and a standard deviation of .094. It has a thick lower tail, with a small spike around .9. There are much larger spikes around 1 and 1.1, indicating that many PHAs set their payment standard around 100 % or 110 % of FMR.

Exhibit 4: Distribution of r=Payment Standard/FMR



Note: FMR=Fair Market Rent. Source: author's calculations using MTCS/PIC data for 2000-2009.

The Payment Standard equals r multiplied by FMR:

$$\text{Payment Standard} = r*\text{FMR} \qquad \text{(eq7)}$$

Substituting eq7 into eq5:

$$\text{HAP} = r*\text{FMR} - \text{H1} - .3*\text{income} \qquad \text{(eq8)}$$

Substituting eq8 into eq1:

$$\text{burden} = .3 + \text{H1} + \text{rent}/\text{income} - r*\text{FMR}/\text{income} \qquad \text{(eq9)}$$

Actual rent at the 40th (or sometimes 50th) percentile of the rent distribution in the FMR region for a given number of bedrooms (FMR*) equals HUD's measure (FMR) plus measurement error (error):

$$FMR^* = FMR + error \quad \text{(eq10)}$$

Gross rent is FMR* plus a household preference factor, H2, for a unit with gross rent above or below FMR*:

$$rent = H2 + FMR^* \quad \text{(eq11)}$$

Substituting eq10 into eq11:

$$rent = H2 + FMR + error \quad \text{(eq12)}$$

Substituting eq12 into eq9:

$$Burden = .3 + H1/income + H2/income + FMR/income - r^*FMR/income + error/income \quad \text{(eq13)}$$

Simplifying:

$$Burden = .3 + (H1 + H2)/income + FMR(1-r)/income + error/income \quad \text{(eq14)}$$

where H1>=0, and (H1 + H2)/income + FMR(1-r)/income + error/income is nonnegative as well. That is, rent burden can't fall below 30 %.

So rent burden can modeled as function of three main factors:

A) Household housing preferences. In particular, tendencies to select units with gross rent 1) below the Payment Standard, and/or 2) above or below rent at the actual 40th (or sometimes 50th) percentile of the rent distribution. Note that the incentive to select units renting below the Payment Standard diminished after 1998 when doing so no longer reduced rent burden.

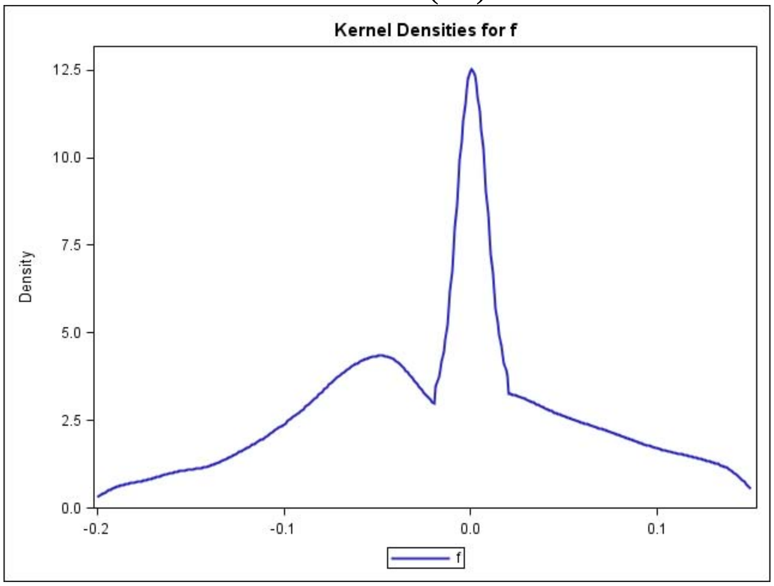
Rent burden increases significantly with home ratings (see Exhibit 3), suggesting that HCVP families are willing to incur higher burdens in order to live in better housing.

B) FMR(1-r)/income. In words, rent burden increases with FMR interacted with 1 minus the ratio of Payment Standard to FMR, relative to household income.

Exhibit 5 depicts the distribution of f=FMR(1-r)/income, which has a mean and median of 0, and standard deviation of .094. For the average household f is 0, thus not affecting rent burden.

Exhibit 6 reports percentages of families with excess burdens of at least 41 % according to f=FMR(1-r)/income in three categories: less than or equal to -.045; -.044 to .013, and .014 or above. The cut points are approximately the 33rd and 66th percentiles. The fraction of households with excess burdens is slightly higher for the lower third (6.783 %) of the distribution compared to the middle third (6.316 %). There is a large increase in burden for families in the upper third of the distribution. 12.371 % of households with f>=.014 have burdens of at least 41 %.

Exhibit 5: Distribution of f=FMR(1-r)/income

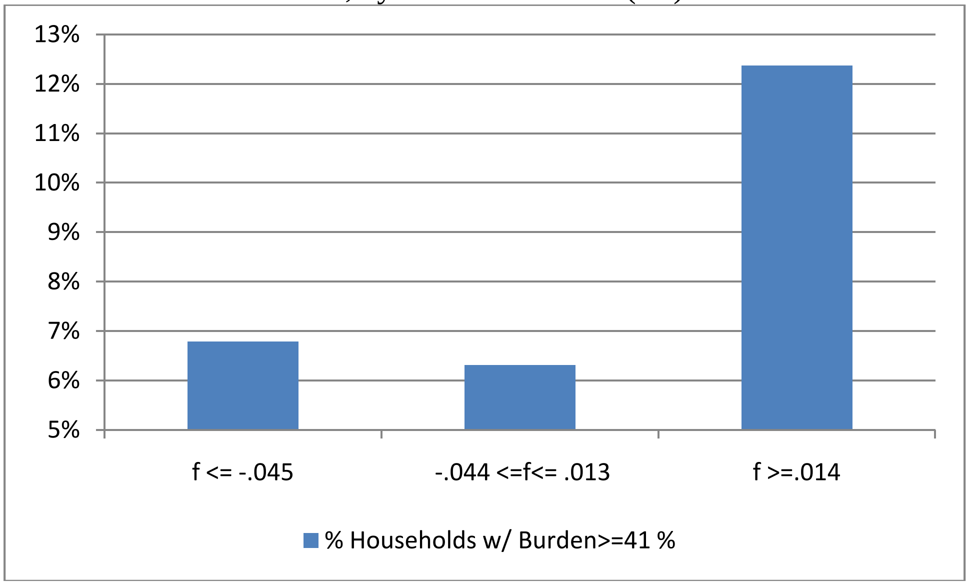


Note: FMR=Fair Market Rent, r=Payment Standard/FMR, and income is household income adjusted per HUD guidelines. Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

C) Deviations in FMR from actual 40th or 50th percentile rents, relative to household income. While Fair Market Rent may be measured with error, the error is difficult to quantify. It is likely that measurement error is much lower in decennial census years, when more data are available on rents by bedrooms for small geographic areas. In other years, census estimates are updated via random digit dialing. When the American Community Survey is fully implemented, it will provide intercensal estimates which may improve FMR measurement.

FMR regions are fairly large (typically counties), and rents vary within FMR regions. Even if FMR is measured well at the FMR region level, this doesn't necessarily imply that FMR is a good measure of neighborhood rents.

Exhibit 6: Burdens>=41 %, by Levels of f=FMR(1-r)/income



Note: FMR=Fair Market Rent, r=Payment Standard/FMR, and income is household income adjusted per HUD guidelines. Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

IV The Data

Data are extracted from HUD's MTCS/PIC data system. The system has quarterly entries for each family receiving rental assistance starting in 1995. Data are available on rent burden and a large number of other household characteristics.

For the remainder of the study, I explore data on families admitted to the HCVP program between 2000 and 2009. Families admitted before 2000 are excluded due to the rapid decline in rent burden from 1995-1999 (see Exhibits 1 and 2).

I exclude Certificate households because 1) their rent burden regulations differ from HCVP regulations, and 2) there are few Certificates today, so the program is of little policy importance. I also exclude voucher households in the Moving To Work demonstration program, due to discretion in rent burden determination and uneven reporting. Homeownership vouchers are also excluded.

I also drop some outlier observations with suspect data. I exclude households if: 1) adjusted annual income is below $250 or above $40,000; or 2) gross rent (including utilities) is above $3,000 a month; or 3) rent burden is less than 28 % or more than 90 %. I also exclude households with multiple admissions.

The data system is transaction based. The most common transactions are 1) admissions; 2) annual re-exams; 3) interim re-exams due to changes in eligibility factors such as income or family size; 4) moves; and 5) exits from the program. The system captures the most recent transaction at the end of each quarter. If multiple transactions for a household occur during a quarter, only the most recent is available. If there is no transaction during a quarter, the family's entry is a duplicate of the entry for the previous quarter.

Rent contracts are effective for one year, and most households have only one transaction per year. Therefore most changes in rent burden are annual (not quarterly). Accordingly, for this study I employ longitudinal research files that capture the most recent transaction at the end of each year for each family.

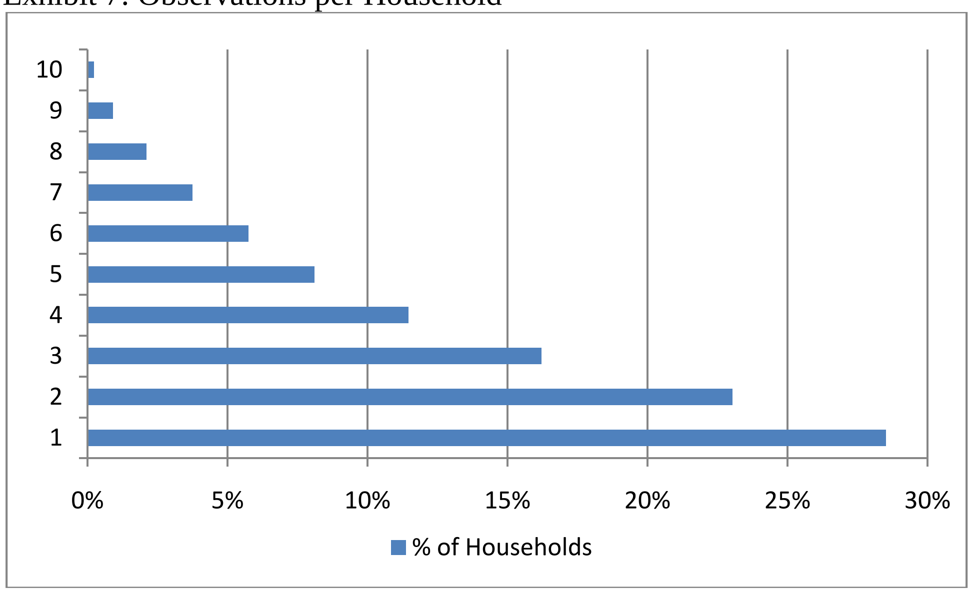
The data provide a consistent snapshot for each family on December 31st of each year. Compared to the quarterly data, the longitudinal data have been edited somewhat to make them more comparable across years. For instance, variable names that change over time are standardized, duplicate records are eliminated, and data items from multiple tables are combined into a single file.

V Summary Statistics

In total, my sample consists of 6,851,746 observations on 1,978,157 HCVP families admitted between 2000 and 2009. Exhibit 7 depicts the distribution of observations per family. The median number of observations per household is 2, and the mean is 2.974. 27.798 % of households have 5 or more observations, and less than 7 % have 7 or more observations.

Exhibit 8 reports mean burden by years in program. Exhibit 9 reports trends in percentages of households with normal rent burdens no more than 31 %; Exhibit 10 reports trends for medium burdens between 30 % and 40 %; Exhibit 11 reports trends for excess burdens of at least 41 %.

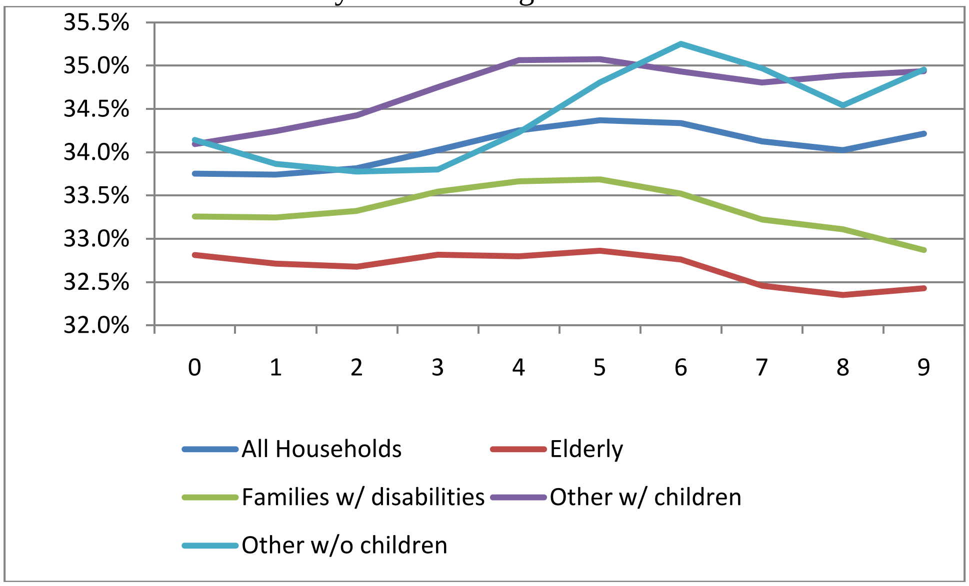
Exhibit 7: Observations per Household



N=6,851,746 observations on 1,978,157 households. Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

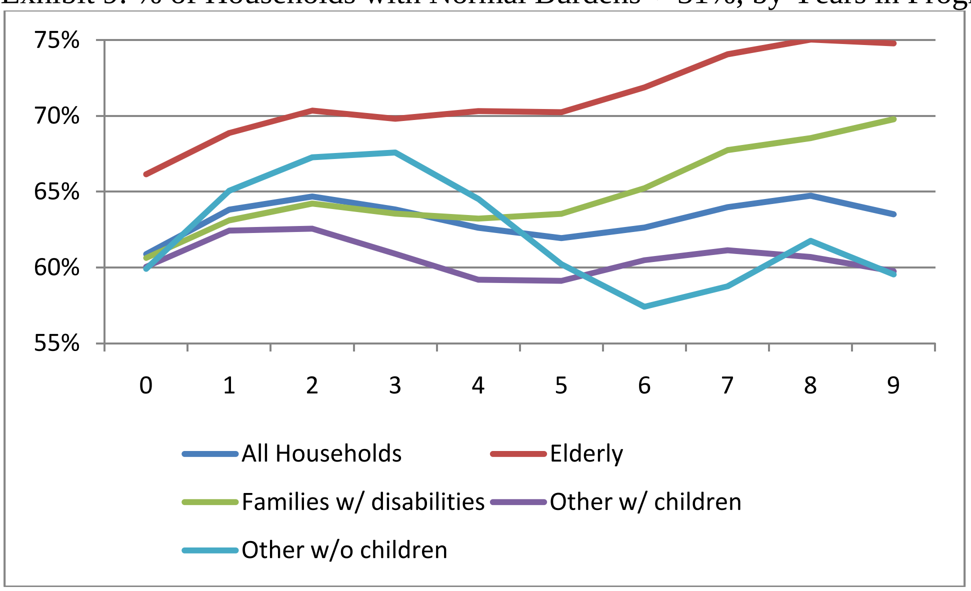
Trends are reported for five household categories: 1) all households; 2) households with an elderly head or spouse (12.609 % of sample); 3) non-elderly households where the head or spouse has a disability (25.370 % of sample); 4) other households with children (45.453 % of sample); and 5) other households without children (16.568 % of sample).

Exhibit 8: Mean Burden by Years in Program



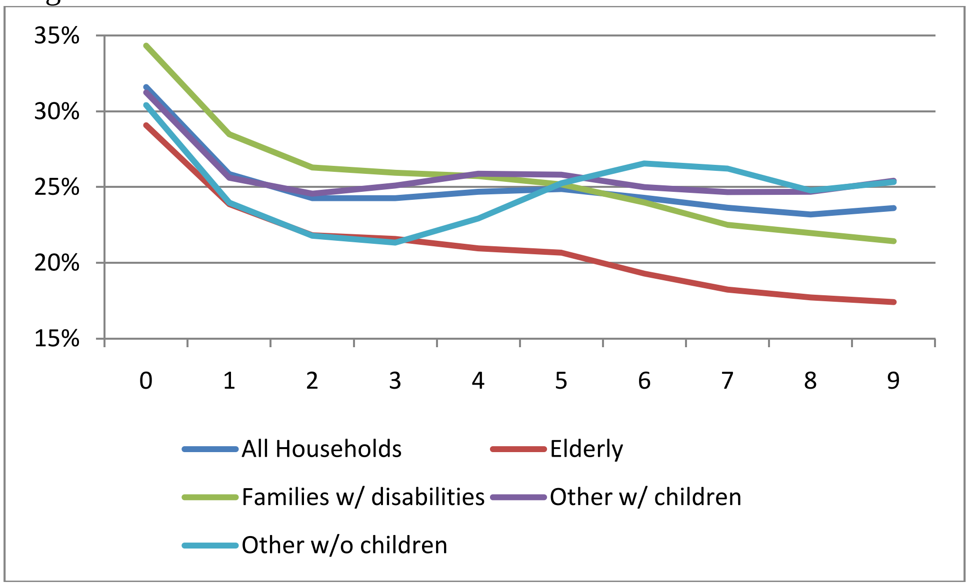
N=6,851,746. Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 9: % of Households with Normal Burdens<=31%, by Years in Program



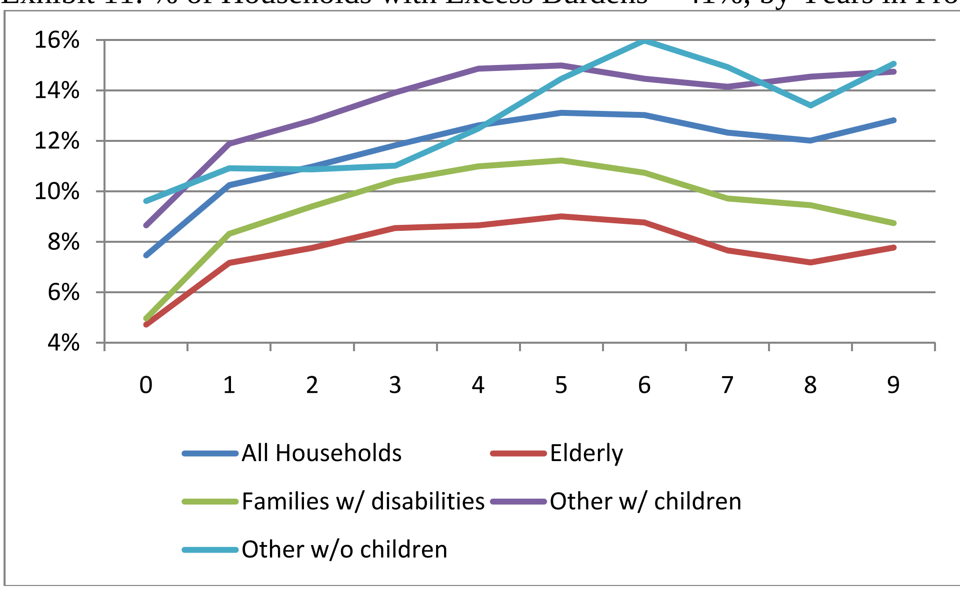
N=6,851,746. Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 10: % of Households with Medium Burdens between 32% and 40%, by Years in Program



Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 11: % of Households with Excess Burdens>=41%, by Years in Program



Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Mean burdens are lowest for elderly families, and highest for the largest category - 'other families with children'. Patterns of burdens are similar for all household categories. In the year of admittance (year 0), burdens are relatively low. This is not surprising, given that initially burdens are not supposed to exceed 40 %. Note that data for year 0 are for the last record for the family in their admission year. For about 26 % of households, the admission year record is not for their admission date (see Exhibits 1 and 2 for burden data on admission dates).

For several years after admission, mean burden increases, along with the percentage of families with excess burdens; the share of families with medium burdens decreases. At the end of the admission year, the mean burden for all households is 33.752 % of income, and the share of households with medium burdens is 31.607 %. The percentage of all households with excess burdens of at least 41 % is 7.465 % in the admission year. By year 4, mean burden is 34.251 %, the share of households with medium burden is 24.695 %, and the share with excess burdens is 12.612 %.

The pattern of normal burdens below 32 % is contrary to the mean for the first several years. While the mean increases after admission, so does the share with normal burdens. For the admission year, the mean burden for all households is 33.752 % of income, and the share of all households with normal burden is 60.876 %. After 2 years, although the mean for all households increases to 33.816 %, the share of households with normal burden increases to 64.685 %. In year 3, the share of households with normal burdens starts to decrease, reaching 61.940 % by year 5.

While burdens tend to decrease starting around year 5 or 6, this may be an artifact of rapidly dwindling observations over time (see Exhibit 7).

VI Burden and Exit

Burden and Exit Rates

The main focus of my study is examining how rent burden changes with program tenure. Of course, rent burden might also influence the likelihood of exit from the program.

Numerous studies have examined duration in the HCVP program. Olson et al. (2005) and Cortes et al. (2009) find a negative relationship between duration and gross rent relative to area housing costs. They do not consider the impact of gross rent relative to income, however.

It’s not possible to predict the sign of the effect of burden on exit. Some families might leave sooner if the only rental units available meeting HUD standards are too costly. Yet families with rising incomes may exit because they no longer meet income requirements; these families might have low burdens when leaving the HCVP program.

A Markov Chain model isn’t appropriate for examining the relationship between burden and exit if exit rates vary with years in program for reasons unrelated to burden. This would violate the assumption of fixed transition probabilities over time.

To explore exit and burden, I estimate logistic regression model $y_{ij}=\alpha_{ij} + e_{ij}$ where y is a binary indicator for exit, α is an intercept shift, e is a random error term, i is years in the HCVP, and j is the families rent burden category. i equals 0 in the admission year and is truncated at 6. j=1 for normal ($\leq$31 %), 2 for medium (32-40 %), and 3 for excess burden ($\geq$41 %).

For brevity, for the remainder of the study I limit my analysis to non-elderly households at admission, with children, where neither the head nor spouse has a disability (the ‘other families with children’ category discussed above). I focus on this subset because they are the largest family category and tend to have the greatest burdens.

Families with multiple admissions are excluded. I also drop families with two or more consecutive years of missing data. Exhibit 12 reports households analyzed by year. Counts are reported for new admissions, other households, and all households. In 2000 there are 26,171 families, all of which are new program participants. By 2009, there are 366,370 households and 75,022 new admissions. In total, I analyze 2,470,944 observations on 826,080 families.

Exhibit 12: Households Analyzed by Year

Year	New Admissions	Other Households	All Households
2000	26171	0	26171
2001	91511	2854	94365
2002	102776	72169	174945
2003	100729	145193	245922
2004	77495	192798	270293
2005	67635	215766	283401
2006	91067	215449	306516
2007	100238	244554	344792
2008	93436	264733	358169
2009	75022	291348	366370
Total	826080	1644864	2470944

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 13 reports logistic regression odds ratio estimates, where the reference category is families with normal burden in their admission year. All of the effects except for excess burden in year 5 have coefficients significant at the .001 level. Mean predicted probabilities of exit are plotted in Exhibit 14.

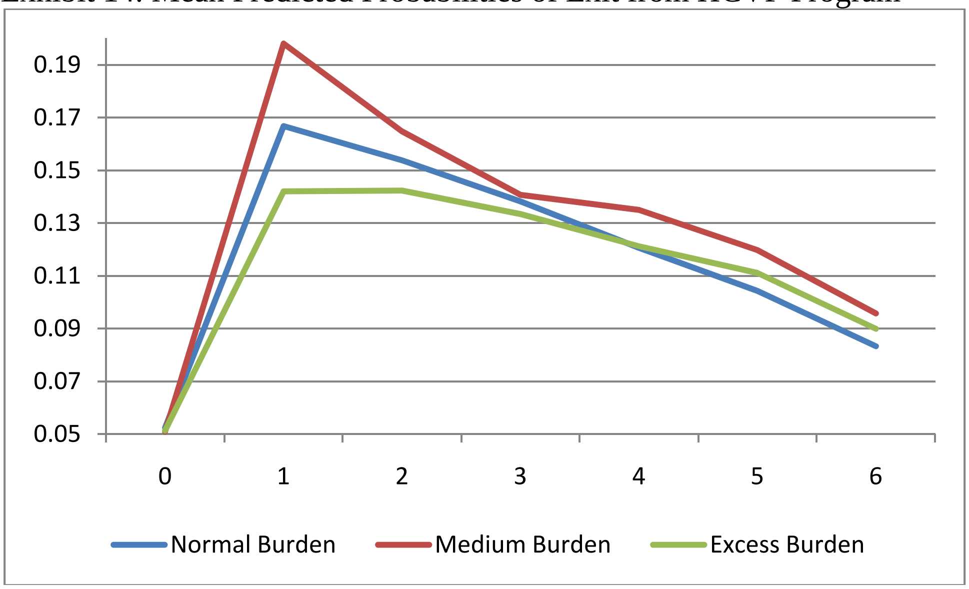
Exhibit 13: Logistic Regression Odds Ratio Estimates

Years in program	Burden category	Odds ratio estimate	Lower 95 % CI	Upper 95 % CI
0	Medium	0.971	0.951	0.993
0	Excess	0.980	0.946	1.016
1	Normal	3.625	3.570	3.681
1	Medium	4.474	4.393	4.555
1	Excess	2.999	2.924	3.076
2	Normal	3.292	3.238	3.348
2	Medium	3.573	3.499	3.649
2	Excess	3.006	2.923	3.091
3	Normal	2.905	2.850	2.961
3	Medium	2.965	2.891	3.041
3	Excess	2.788	2.697	2.881
4	Normal	2.484	2.429	2.540

4	Medium	2.827	2.743	2.913
4	Excess	2.499	2.400	2.601
5	Normal	2.109	2.053	2.166
5	Medium	2.464	2.375	2.557
5	Excess	2.263	2.153	2.378
6	Normal	1.646	1.603	1.691
6	Medium	1.917	1.846	1.992
6	Excess	1.789	1.699	1.884

N=2,470,944, -2 log likelihood=1689606.7. Reference category is normal burden in the admission year, and years in program is truncated as 6. Source: author's calculations using MTCS/PIC data for 2000-09 admissions.

Exhibit 14: Mean Predicted Probabilities of Exit from HCVP Program



N=2,470,944. Years in program is truncated at 6. Source: author's calculations using MTCS/PIC data for 2000-09 admissions.

The probability of exit varies widely with duration regardless of rent burden. Exit hazards are lowest in the admission year, and highest in the first year past admission. Exit probabilities tend to decline each remaining year.

Prior research indicates factors such as housing preferences (Olson et al. 2005) and number of children (Cortes et al. 2009) may play import roles determining exit. Because of large variation in exit probabilities with duration, exit is an unlikely candidate as a Markov Chain.

In the admission year, there is little difference in exit chances across burden categories. Families with normal burden have an estimated .052 exit probability, compared to .051 for families with medium or excess burden.

In each year past admission, households with medium burden are most likely to exit. In year 1, exit probabilities are .167 for normal burden, .198 for medium burden, and .142 for excess burden families. By year 6, exit chances have declined substantially - .083 for normal, .096 for medium, and .090 for excess burden households.

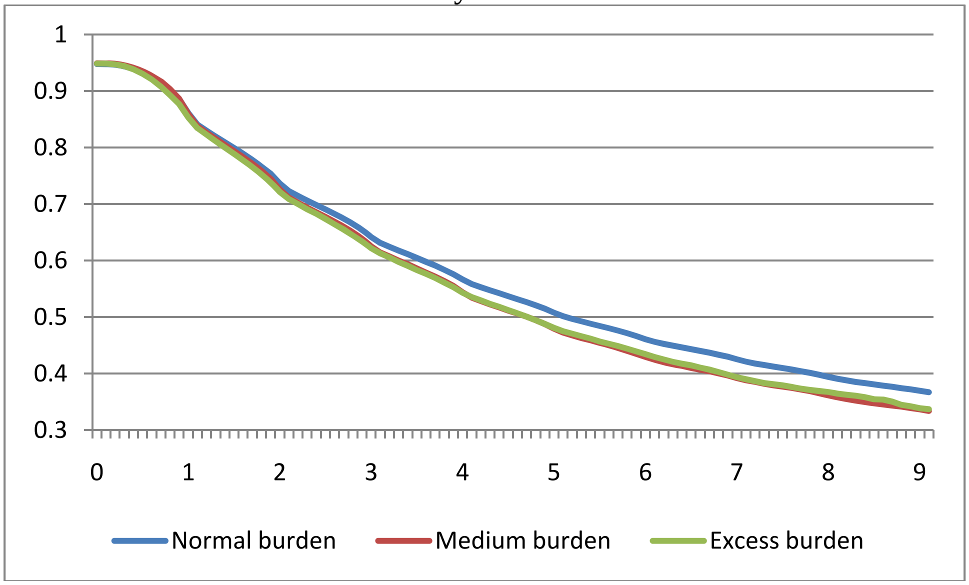
Exit Rates and the Markov Chain Model
The Markov Chain model in the next section predicts burden for households remaining in the program. Changing exit rates are not a problem for the Markov Chain model if the proportional hazard of exiting is constant across burden categories. Yet if the proportion of households exiting varies with burden over time, this would lead to changing transition probabilities violating the stationarity assumption.

Odds ratios vary greatly the first couple of years in the program. Starting in year 2, proportional exit rates are fairly stable over time. Differences in proportional exit chances between early and later years suggest that a Markov chain model needs separate states for families entering the program.

Burden and Program Tenure
To explore the effect of burden on time in the HCVP program, I estimate a Kaplan-Meier survival function stratified by burden category at admission. The survival function estimates the probability of remaining in the HCVP program over time. The estimated survival curve is depicted in Exhibit 15.

Households with normal burden in their admission year have the greatest estimated time in program, and there is little difference between households admitted with medium or excess burden. Median survival time is 5.131 years for households admitted with normal burden, compared to 4.715 for those with medium burden and 4.728 for those with excess burden. The difference in median tenure between households admitted with normal burden and other households is about .4 years.

Exhibit 15: Survival Probabilities by Burden at Admission



Source: author's calculations using MTCS/PIC data for 2000-09 admissions.

VII Markov Chain Model

Data

In this section, I model HCVP rent burden as a discrete time stochastic process. The sample is the same group of families with children used in the previous section (see Exhibit 12). While household status can change over time, this is just one factor among many possibly affecting rent burden.

Assumptions

I assume a household's burden falls into 3 categories, normal, medium, and excess:

A) Normal burden -- rent burden ≤ 31 % ;
B) Medium burden -- 32 % ≤ rent burden ≤ 40 %;
C) Excess burden -- rent burden ≥ 41 %.

I define 12 states based on rent burden in the prior and current year:

A -- Normal burden at admission (no burden in the prior year);
AA -- Normal burden in the prior and current year;
AB -- Normal burden in the prior year, medium burden in the current year;
AC -- Normal burden in the prior year, excess burden in the current year;
B -- Medium burden at admission (no burden in the prior year);
BA -- Medium burden in the prior year, normal burden in the current year;
BB -- Medium burden in the prior and current year;
BC -- Medium burden in the prior year, excess burden in the current year;
C -- Excess burden at admission (no burden in the prior year);
CA -- Excess burden in the prior year, normal burden in the current year;
CB -- Excess burden in the prior year, medium burden in the current year;
CC -- Excess burden in the prior and current year.

Current states are known. Burden in subsequent years satisfies the following assumption: the conditional distribution of any future state X_{n+1} is independent of past states, depending only on the current state X_n. That is, when the household is in state i, there is a fixed probability P_{ij} of being in state j in the next year.

Under these conditions, rent burden is a Markov Chain. Let the current year be denoted by year 0. For households in their admission year, rent burden in year 1 depends only on their current burden. For other households, burden in year 1 depends on rent burden in the current and prior year.

Because of differences in burden and proportional exit rates early in the program, the stationarity assumption is unlikely to hold making no distinction between burden at admission and in subsequent years. Thus the model includes three separate states solely for admission (A, B, and C). Modeling rent burden in years 2 and above as dependent on multiple periods also makes the stationarity assumption more plausible. Therefore the remaining nine states are based on two years of data.

Transition Probabilities

Exhibits 16-19 report transition probability matrices P^n for years n=1 to 4, respectively. The matrices report the conditional probability Pij of being in state j in n years, given current state i. There is a row for each possible current state i, and a column for each possible state j in n years.

P^1 was calculated by cross tabulating states with states lagged one year. Due to stationarity, the transition matrices for years n>1 can be derived by raising P^1 to the nth power: $P^2 = P^1$ squared, $P^3 = P^1$ cubed, and $P^4 = P^1$ to the 4th power. While the matrices used for calculations are square (12x12), 3 columns contain all zeros- households cannot transition to the admission states A, B, and C. Thus I report 12x9 matrices excluding nuisance columns for admission states.

Exhibit 16: P^1 Transition Probability Matrix

State	AA	AB	AC	BA	BB	BC	CA	CB	CC
A	0.853	0.105	0.042	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.850	0.110	0.040	0.000	0.000	0.000	0.000	0.000	0.000
AB	0.000	0.000	0.000	0.356	0.503	0.140	0.000	0.000	0.000
AC	0.000	0.000	0.000	0.000	0.000	0.000	0.313	0.245	0.442
B	0.000	0.000	0.000	0.291	0.546	0.163	0.000	0.000	0.000
BA	0.760	0.179	0.061	0.000	0.000	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.000	0.279	0.577	0.144	0.000	0.000	0.000
BC	0.000	0.000	0.000	0.000	0.000	0.000	0.187	0.339	0.474
C	0.000	0.000	0.000	0.000	0.000	0.000	0.244	0.255	0.501
CA	0.748	0.159	0.093	0.000	0.000	0.000	0.000	0.000	0.000
CB	0.000	0.000	0.000	0.269	0.536	0.195	0.000	0.000	0.000
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.173	0.255	0.571

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 17: P^2 Transition Probability Matrix

State	AA	AB	AC	BA	BB	BC	CA	CB	CC
A	0.725	0.094	0.034	0.037	0.053	0.015	0.013	0.010	0.019
AA	0.722	0.093	0.034	0.039	0.055	0.015	0.013	0.010	0.018
AB	0.271	0.064	0.022	0.140	0.290	0.073	0.026	0.048	0.067
AC	0.234	0.050	0.029	0.066	0.131	0.048	0.077	0.113	0.252
B	0.221	0.052	0.018	0.152	0.315	0.079	0.030	0.055	0.077
BA	0.646	0.083	0.031	0.064	0.090	0.025	0.019	0.015	0.027
BB	0.212	0.050	0.017	0.161	0.332	0.083	0.027	0.049	0.069
BC	0.140	0.030	0.017	0.091	0.181	0.066	0.082	0.121	0.271
C	0.183	0.039	0.023	0.069	0.136	0.050	0.087	0.128	0.286
CA	0.636	0.082	0.030	0.057	0.080	0.022	0.029	0.023	0.041
CB	0.205	0.048	0.016	0.149	0.309	0.077	0.036	0.066	0.093
CC	0.130	0.028	0.016	0.069	0.137	0.050	0.099	0.146	0.326

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 18: P^3 Transition Probability Matrix

State	AA	AB	AC	BA	BB	BC	CA	CB	CC
A	0.654	0.088	0.033	0.051	0.083	0.023	0.017	0.018	0.033
AA	0.653	0.088	0.033	0.051	0.084	0.023	0.017	0.018	0.033
AB	0.356	0.059	0.022	0.116	0.225	0.060	0.032	0.047	0.082
AC	0.306	0.050	0.021	0.085	0.161	0.048	0.062	0.088	0.180
B	0.327	0.056	0.021	0.121	0.237	0.064	0.034	0.051	0.089
BA	0.612	0.085	0.032	0.059	0.102	0.028	0.019	0.023	0.041
BB	0.323	0.056	0.021	0.124	0.243	0.065	0.033	0.050	0.086
BC	0.250	0.045	0.019	0.094	0.184	0.054	0.065	0.096	0.194
C	0.272	0.046	0.020	0.086	0.167	0.050	0.066	0.096	0.197
CA	0.605	0.084	0.032	0.058	0.100	0.028	0.021	0.025	0.047
CB	0.315	0.055	0.021	0.121	0.238	0.064	0.036	0.054	0.097
CC	0.236	0.042	0.019	0.087	0.171	0.052	0.071	0.104	0.217

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 19: P^4 Transition Probability Matrix

State	AA	AB	AC	BA	BB	BC	CA	CB	CC
A	0.607	0.084	0.031	0.060	0.102	0.028	0.020	0.024	0.044
AA	0.607	0.083	0.031	0.060	0.103	0.028	0.020	0.024	0.044
AB	0.415	0.065	0.024	0.096	0.185	0.050	0.032	0.047	0.085
AC	0.371	0.059	0.023	0.086	0.165	0.047	0.047	0.067	0.135
B	0.395	0.063	0.024	0.100	0.192	0.052	0.034	0.050	0.091
BA	0.579	0.081	0.030	0.065	0.114	0.031	0.022	0.028	0.051
BB	0.393	0.063	0.024	0.101	0.195	0.053	0.034	0.049	0.089
BC	0.332	0.054	0.022	0.093	0.180	0.052	0.050	0.072	0.145
C	0.346	0.056	0.022	0.089	0.171	0.049	0.050	0.072	0.145
CA	0.574	0.080	0.030	0.065	0.114	0.031	0.023	0.029	0.054
CB	0.386	0.062	0.023	0.100	0.194	0.053	0.035	0.052	0.095
CC	0.320	0.053	0.021	0.091	0.176	0.051	0.053	0.078	0.157

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Suppose a family is currently in state B: medium burden at admission. P^1 indicates a .291 chance of normal burden (state BA) next year. The probability of medium burden (state BB) next year is .546, and the chance of excess burden (state BC) is .163.

Burden tends to increase with program tenure. Suppose a family is currently in state A (normal burden at admission). Their chance of excess burden increases monotonically with time. In year 1, their probability of excess burden (state AC) is .042. P^2 indicates their probability of excess burden in year 2. The family could transition from A to excess burden in year 2 in three possible ways (states AC, BC, or CC) depending on their burden in year 1. The probability of AC or BC or CC in year 2 equals .034 + .015 + .019=.068. In year 3, their probability of excess burden is .089, and in year 4 it increases to .103.

While there is a trend toward increasing burden, there is also considerable mobility across states over time. For a family with excess burden at admission (state C), their chance of normal or medium burden increases monotonically with time. In year 1, their probability of normal or medium burden (state CA or CB) is .244 + .255=.499. Their chance of normal or medium burden in year 2 (state AA, AB, BA, BB, CA, or CB) is .641, and in year 3 it increases to .733. By year 4 their probability of normal or medium burden is .783.

Multiple Years of Excess Burden

Suppose a family is in any given current state. What's their probability of exactly 2 periods of excess burden over the next 4 years? There are $\binom{4}{2}$ =6 possible ways this could happen: $\{B_1B_2, B_1B_3, B_1B_4, B_2B_3, B_2B_4, B_3B_4\}$, where B_i=excess burden in year i. While the events are not independent, the probability can be computed with the inclusion/exclusion formula.[4]

Exhibit 20 reports probabilities of 0-4 years of excess burden during the next 4 years for each current state. Suppose a family has had excess burden this year and last (state CC). Over the next 4 years there is a .383 chance of 1 year, a .339 chance of 2 years, and a .120 chance of 3 years of excess burden. The probabilities decrease with burden at admission. For families in state AA, the corresponding probabilities are .240, .029, and .001, respectively. The probability of 4 years excess burden is extremely low for all households.

Exhibit 20: Probability of Multiple Years of Excess Burden, Next 4 Years

Current state	Probability of 0 years excess burden	Probability of 1 year excess burden	Probability of 2 years excess burden	Probability of 3 years excess burden	Probability of 4 years excess burden
A	0.730	0.240	0.029	0.001	0.000
AA	0.731	0.239	0.028	0.001	0.000
AB	0.506	0.376	0.104	0.013	0.001
AC	0.224	0.418	0.275	0.076	0.007
B	0.476	0.388	0.119	0.016	0.001
BA	0.688	0.270	0.039	0.002	0.000
BB	0.491	0.382	0.111	0.014	0.001
BC	0.194	0.407	0.298	0.091	0.010
C	0.184	0.405	0.306	0.095	0.010
CA	0.650	0.296	0.050	0.004	0.000
CB	0.444	0.400	0.135	0.020	0.001
CC	0.143	0.383	0.339	0.120	0.015

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

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[4] According to the inclusion/exclusion formula, the "the probability of the union of n events equals the sum of the probabilities … taken one at time … minus the sum of the probabilities … taken two at a time plus the sum of the probabilities … taken three at a time, and so on" (Ross 2007, p9).

Exhibit 21 reports probabilities of 2-4 years of consecutive excess burden over the next four years for each current state. For any given number of years of excess burden, the probability of consecutive years is less than or equal to the probability for multiple years. For instance, given medium burden at admission (state B), the chance of 2 years of excess over the next 4 years is .119, compared to a .058 chance of 2 consecutive years.

Exhibit 21: Probability of Consecutive Years of Excess Burden, Next 4 Years

Current state	Probability of 2 consecutive years excess burden	Probability of 3 consecutive years excess burden	Probability of 4 consecutive years excess burden
A	0.014	0.001	0.000
AA	0.014	0.001	0.000
AB	0.051	0.007	0.001
AC	0.152	0.038	0.007
B	0.058	0.008	0.001
BA	0.019	0.001	0.000
BB	0.054	0.008	0.001
BC	0.165	0.046	0.010
C	0.173	0.048	0.010
CA	0.026	0.002	0.000
CB	0.067	0.010	0.001
CC	0.197	0.061	0.015

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Predictions

To gauge the model's predictive power, Exhibit 22 reports actual and predicted household shares with normal, medium, and excess burden for 4 years past admission. Absolute percentage differences between actual and predicted outcomes are also reported. Predicted outcomes X_n equals X_0P^n, where X_0 is a row vector of current household shares in each state, and P^n is the year n transition probability matrix.

Exhibit 22: Actual and Predicted Outcomes for 2000-02, 2003-05, and 2000-05 Admissions

2000-2002 Admissions

	Normal Burden			Medium Burden			Excess Burden		
Year	Actual	Predicted	Absolute % difference	Actual	Predicted	Absolute % difference	Actual	Predicted	Absolute % difference
0	0.640			0.278			0.082		
1	0.678	0.647	4.707%	0.223	0.240	7.033%	0.099	0.114	14.019%
2	0.669	0.636	5.084%	0.222	0.242	9.015%	0.109	0.121	10.657%
3	0.621	0.631	1.469%	0.246	0.242	1.600%	0.132	0.127	4.085%
4	0.574	0.626	8.753%	0.270	0.243	10.453%	0.156	0.130	17.973%

2003-2005 Admissions

	Normal Burden			Medium Burden			Excess Burden		
Year	Actual	Predicted	Absolute % difference	Actual	Predicted	Absolute % difference	Actual	Predicted	Absolute % difference
0	0.593			0.320			0.087		
1	0.585	0.620	5.890%	0.281	0.259	8.134%	0.135	0.121	10.581%
2	0.573	0.618	7.546%	0.276	0.254	8.004%	0.151	0.127	17.090%
3	0.593	0.619	4.222%	0.260	0.249	4.132%	0.147	0.131	10.847%
4	0.631	0.619	1.908%	0.240	0.248	3.204%	0.129	0.133	3.120%

2000-2005 Admissions

	Normal Burden			Medium Burden			Excess Burden		
Year	Actual	Predicted	Absolute % difference	Actual	Predicted	Absolute % difference	Actual	Predicted	Absolute % difference
0	0.615			0.300			0.084		
1	0.625	0.632	1.188%	0.256	0.250	2.585%	0.119	0.117	1.635%
2	0.619	0.626	1.220%	0.250	0.248	0.638%	0.131	0.124	5.490%
3	0.607	0.624	2.788%	0.253	0.246	3.043%	0.140	0.129	7.866%
4	0.604	0.622	2.955%	0.254	0.246	3.562%	0.142	0.132	7.472%

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Three cohorts of admissions are considered: 2000-02, 2003-05, and 2000-05 (2005 is the last admission year with 4 years of subsequent data). If the assumption of fixed probabilities is valid, the model should predict equally well regardless of program tenure or calendar year.

In general, predictions are closest for the largest burden category (normal) and worst for the smallest category (excess). For 2000-05 for instance, the maximum difference for normal burden predictions is 2.955 %, compared to 3.562 % for medium and 7.866 % for excess burden. For 2000-05 the model slightly over-predicts the share of households with normal burden, and under-predicts shares with medium and excess burden.

While model accuracy varies with program tenure, the evidence varies by study period. For 2000-05 admissions, the model predicts better in earlier years. For instance, the excess burden prediction for 2000-05 admissions is off 1.635 % in year 1, compared to 7.472 % in year 4.

For 2003-05 admissions, predictions become better over time. For instance, the medium burden prediction is off 8.134 % for year 1, compared to 3.324 % in year 4.

One possible reason for differences with program tenure may slightly varying proportional exit rates between year 1 and later years (see Exhibit 14). Differences by program tenure might be mitigated with states based on more years. Of course there is a trade-off between model simplicity and accuracy. Defining states based on up to 3 years of data would require 27 more states (AAA, AAB, … ,CCC), and the transition matrices would increase from 12x12 to 39x39. Furthermore, there are far fewer households with 3 years of data than 2. Thus the probabilities would be more susceptible to outliers.

There is also evidence that the transition probabilities vary with calendar year - the model predicts better for 2000-05 compared to the two shorter time periods. For example, the year 2 prediction for medium burden is off .638 % for the combined sample, compared to 9.015 % for 2000-02, and 8.004 % for 2003-05.

The model also tends to predict slightly better for 2003-05 compared to 2000-02. For instance, the maximum difference in normal burden predictions is 8.753 % for 2000-02, compared to 7.546 % for 2003-05.

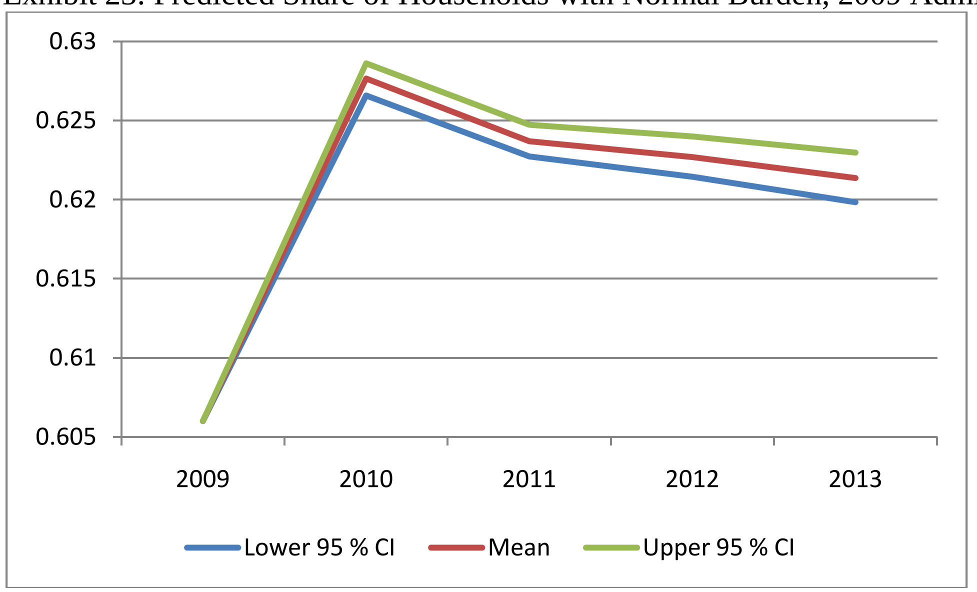
If the model is used to make predictions for recent cohorts, evidence suggests it might perform better excluding the earliest years of data. Another possibility for improving current predictions would be calculating transition matrices by Bayesian updating, giving more recent years of data greater weight.

Variance

One limitation of the Markov Chain model is that it doesn't estimate variance. In this section I demonstrate a method for doing so by re-sampling the data with replacement (bootstrapping). Bootstrapping provides a convenient method for computing confidence intervals directly from percentiles of the sampling distribution of the mean (Lohr 2007, p. 307).

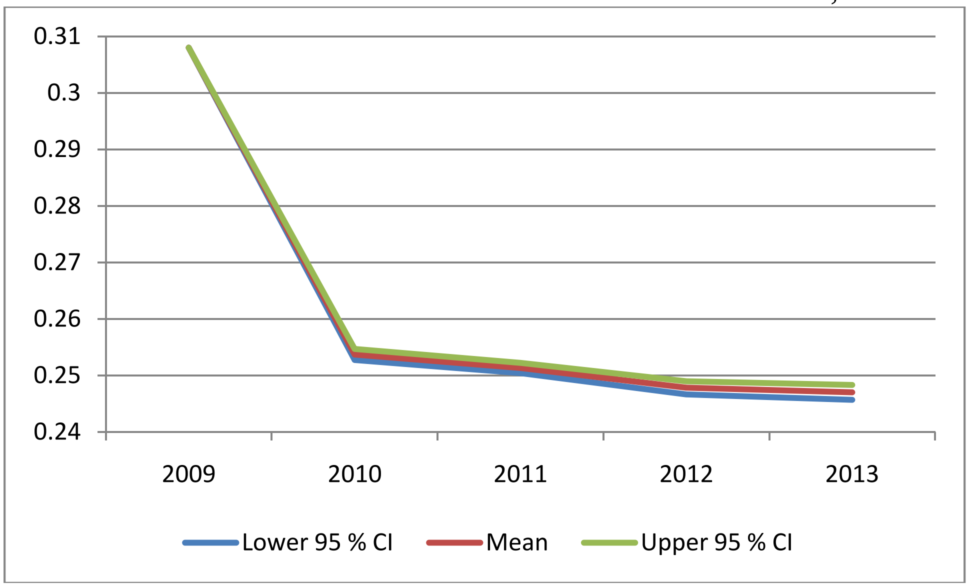
I took 1000 100 % random samples with replacement, generating new transition matrices with each iteration. Exhibit 23 reports mean predictions for 2009 admissions for the share of households with normal burdens in 2010-13, along with 95 % confidence intervals. Confidence limits are the 2.5th and 97.5th percentiles. Exhibits 24 and 25 report corresponding projections for shares with medium and excess burden, respectively. The admission shares are .606, .308, and .087 for normal, medium, and excess burden, respectively.

Exhibit 23: Predicted Share of Households with Normal Burden, 2009 Admissions



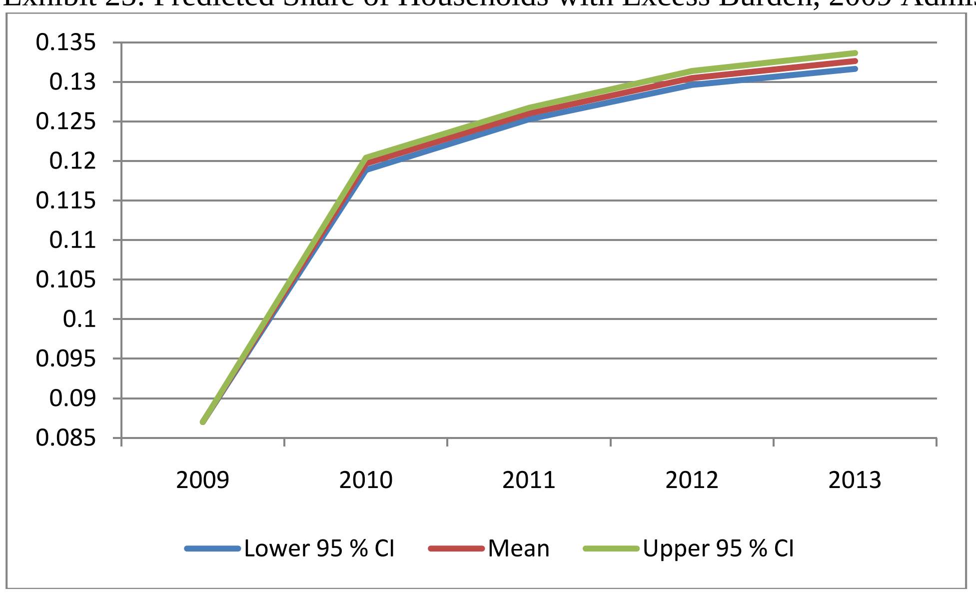
Source: author's calculations using MTCS/PIC data. Transition probabilities are based on admissions between 2000 and 2009.

Exhibit 24: Predicted Share of Households with Medium Burden, 2009 Admissions



Source: author's calculations using MTCS/PIC data. Transition probabilities are based on admissions between 2000 and 2009.

Exhibit 25: Predicted Share of Households with Excess Burden, 2009 Admissions



Source: author's calculations using MTCS/PIC data. Transition probabilities are based on admissions between 2000 and 2009.

The predicted share of families with normal burden increases in year 2010; the confidence interval varies from .627 to .629. Predictions decrease each remaining year, while variance increases. By 2013, the predicted share with normal burden ranges from .620 to .623.

The predicted share of households with medium burden decreases each year past admission, while the predicted share with excess burden increases. The predicted range with medium (excess) burden is .253-.255 (.119-.120) in 2010, and .250-.252 (.125-.127) in 2011. Rates of change decrease in 2012. By 2013, the medium burden prediction ranges from .246 to .248, while the excess burden prediction varies from .132 to .134.

Policy Analysis

In this section I analyze policy effects of f=FMR(1-r)/income discussed in section III. I define 4 categories based on 2 burden categories and 2 f categories:

A) Normal/medium burden, low/medium f -- burden ≤ 40 %, f ≤ .006;
B) Normal/medium burden, high f -- burden ≤ 40 %, f ≥ .007;
C) Excess burden, low/medium f -- burden ≥ 41 %, f ≤ .006;
D) Excess burden, high f -- burden ≥41 %, f ≥ .007.

The f cutpoint is roughly the 66^{th} percentile. I also define 20 states based on the current year for admissions, and the current and prior year otherwise:

A -- Normal/medium burden and low/medium f at admission;
AA -- Normal/medium burden and low/medium f in the prior and current year;
AB -- Normal/medium burden and low/medium f in the prior year, normal/medium burden and high f in the current year;
.
.
.
DD – Excess burden and high f in the prior and current year.

While transition matrices used for computations contain all possible states, the purpose of the model is not to predict HUD and PHA policy. For examining the impact of f on burden, probabilities conditional on f are more convenient. Exhibit 26 reports the year 1 transition matrix conditional on low/medium f in year 1; Exhibit 27 reports corresponding probabilities conditional on high f values.

Exhibit 26: Year 1 Transition Probabilities Conditional on Low/Medium f

State	AA	AC	BA	BC	CA	CC	DA	DC
A	0.934	0.066	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.952	0.048	0.000	0.000	0.000	0.000	0.000	0.000
AB	0.000	0.000	0.950	0.050	0.000	0.000	0.000	0.000
AC	0.000	0.000	0.000	0.000	0.553	0.447	0.000	0.000
AD	0.000	0.000	0.000	0.000	0.000	0.000	0.714	0.286
B	0.000	0.000	0.945	0.055	0.000	0.000	0.000	0.000
BA	0.955	0.045	0.000	0.000	0.000	0.000	0.000	0.000

BB	0.000	0.000	0.957	0.043	0.000	0.000	0.000	0.000
BC	0.000	0.000	0.000	0.000	0.579	0.421	0.000	0.000
BD	0.000	0.000	0.000	0.000	0.000	0.000	0.751	0.249
C	0.000	0.000	0.000	0.000	0.501	0.499	0.000	0.000
CA	0.861	0.139	0.000	0.000	0.000	0.000	0.000	0.000
CB	0.000	0.000	0.889	0.111	0.000	0.000	0.000	0.000
CC	0.000	0.000	0.000	0.000	0.438	0.562	0.000	0.000
CD	0.000	0.000	0.000	0.000	0.000	0.000	0.587	0.413
D	0.000	0.000	0.000	0.000	0.000	0.000	0.705	0.295
DA	0.903	0.097	0.000	0.000	0.000	0.000	0.000	0.000
DB	0.000	0.000	0.905	0.095	0.000	0.000	0.000	0.000
DC	0.000	0.000	0.000	0.000	0.491	0.509	0.000	0.000
DD	0.000	0.000	0.000	0.000	0.000	0.000	0.621	0.379

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Exhibit 27: Year 1 Transition Probabilities Conditional on High f

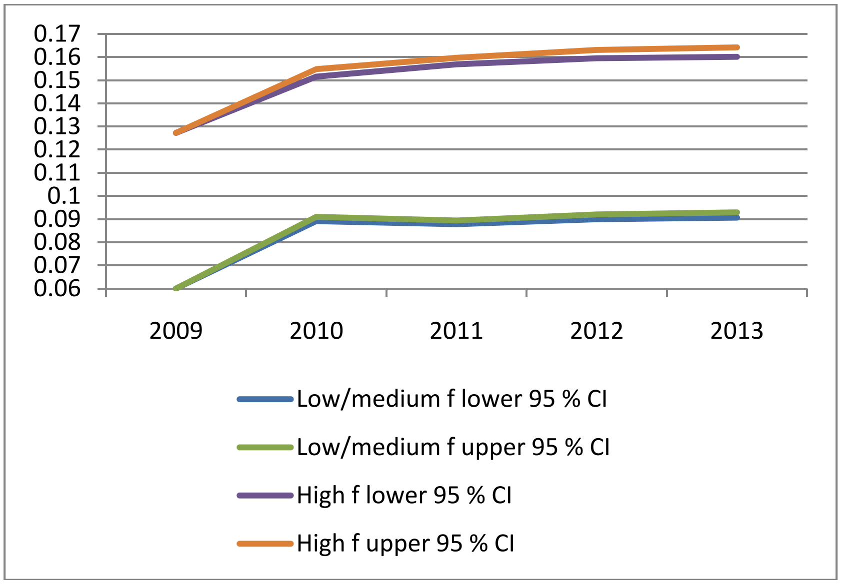
State	AB	AD	BB	BD	CB	CD	DB	DD
A	0.872	0.128	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.872	0.128	0.000	0.000	0.000	0.000	0.000	0.000
AB	0.000	0.000	0.919	0.081	0.000	0.000	0.000	0.000
AC	0.000	0.000	0.000	0.000	0.451	0.549	0.000	0.000
AD	0.000	0.000	0.000	0.000	0.000	0.000	0.443	0.557
B	0.000	0.000	0.915	0.085	0.000	0.000	0.000	0.000
BA	0.906	0.094	0.000	0.000	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.937	0.063	0.000	0.000	0.000	0.000
BC	0.000	0.000	0.000	0.000	0.467	0.533	0.000	0.000
BD	0.000	0.000	0.000	0.000	0.000	0.000	0.512	0.488
C	0.000	0.000	0.000	0.000	0.416	0.584	0.000	0.000
CA	0.775	0.225	0.000	0.000	0.000	0.000	0.000	0.000
CB	0.000	0.000	0.834	0.166	0.000	0.000	0.000	0.000
CC	0.000	0.000	0.000	0.000	0.346	0.654	0.000	0.000
CD	0.000	0.000	0.000	0.000	0.000	0.000	0.356	0.644
D	0.000	0.000	0.000	0.000	0.000	0.000	0.422	0.578
DA	0.795	0.205	0.000	0.000	0.000	0.000	0.000	0.000
DB	0.000	0.000	0.851	0.149	0.000	0.000	0.000	0.000
DC	0.000	0.000	0.000	0.000	0.367	0.633	0.000	0.000
DD	0.000	0.000	0.000	0.000	0.000	0.000	0.371	0.629

Source: author's calculations using MTCS/PIC data for admissions between 2000 and 2009.

Suppose a family is admitted with low/medium burden and high f (state B). In year 1 if f is low/medium they have a .945 chance of normal/medium burden (state BA), and a .055 chance of excess burden (state AB). If f is high they have a .892 chance of normal/medium burden, and a .108 chance of excess burden.

Exhibit 28 depicts 4 year excess burden projections for 2009 admissions conditional on low/medium and high f.

Exhibit 28: Projected Share of Households with Excess Burden Conditional on f, 2009 Admissions

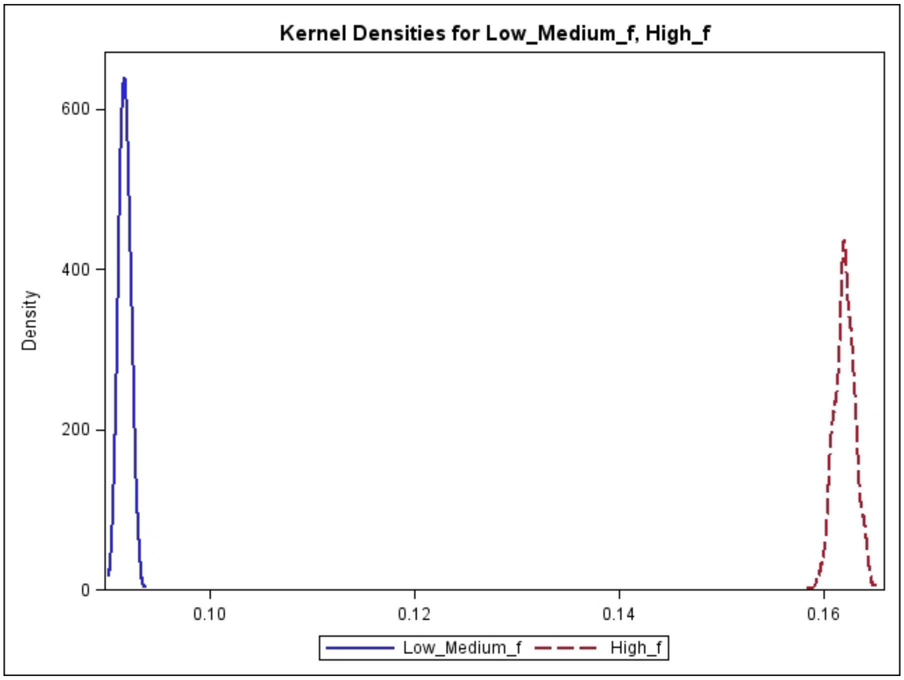


Source: author's calculations using MTCS/PIC data. Projections are for 2009 admissions with transition probabilities based on 2000-09 data.

Although predicted burdens tend to increase after admission regardless of f, families with high f values are much more likely to have excess burden each year. The share of households admitted in 2009 with excess burden is .060 for households with low/medium f, compared to .127 for those with high f. By 2013, the predicted share of households with excess burden is about 75 % higher for those with high f (.160-.164) compared to those with low/medium f (.091-.093).

Exhibit 29 reports bootstrap conditional densities for the predicted share of households in 2013 with excess burden. While bootstrap estimates make it possible to conduct likelihood ratio tests for differences in burden according f, there is virtually no overlap in the conditional distributions. It's obvious that the interaction of HUD and PHA policy relative to income has an effect very large in both magnitude and statistical significance.

Exhibit 29: Bootstrap Densities for 2013 Excess Burden Predictions Conditional on f, 2009 Admissions



Source: author's calculations using MTCS/PIC data. Projections are for 2009 admissions with transition probabilities based on 2000-09 data.

VIII Conclusion

This study models rent burden in the Housing Choice Voucher Program as a Markov Chain. The model predicts rent burden with program tenure, using longitudinal household data for 2000 through 2009.

Results indicate rent burden increases for many years after admission. This is perhaps not surprising, given that restrictions on burden at admission don't apply in following years.

Consistent with results for unassisted low-income families (Martin et al. 2005, Susin 2007), the model also predicts considerable mobility across burden categories over time. For a family with burden over 40 % at admission, their estimated chance of burden below 40 % in their next year is 50 %. Regardless of a family's current burden, the probability of 4 years of burden above 40 % during the next 4 years is very low.

The rent burden formula indicates that HUD policy and PHA policy should not be considered in isolation. Markov Chain estimates imply that the interaction of Fair Market Rent and the Payment Standard have effects large in both magnitude and statistical significance.

One limitation of the Markov Chain model is that it doesn't estimate variance. I demonstrate a simple method for doing so via bootstrapping.

I also examine the relationship between burden and program tenure using survival analysis. Exit probabilities are low in the 1st year in the program, with little differences due to burden. Chances of leaving increase after admission and households with moderate burdens tend to have the greatest exit rates. Households admitted with burdens below 32 % tend to stay in the program the longest.

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